Seismic analysis of structures: Stress-resultant interaction based on response spectra

Análise sísmica de estruturas: Interação de esforços com base em espectros de resposta

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Abstract

The focus of this paper is to explore a methodology that characterizes a structure response to a dynamic excitation more accurately and less conservatively than the one currently adopted in the application of design codes.

For the design of a reinforced concrete section, there is a set of *n* variables that need to be quantified: the axial force, bending moments or displacements. Instead of assuming the simultaneity of all the variables maximum values calculated by a typical modal analysis (using a response spectra and modal combination criteria), the studied method produces a surface in an *n*-dimension space that reproduces the interactions between those variables.

Besides being an interesting theoretical problem, it has clear applications in the design of columns subjected to a combination of axial force with bending moments, leading to significant reductions of steel reinforcement ratios and allowing a material economy without compromising the structural safety.

Resumo

O propósito deste artigo é explorar uma metodologia que caracterize a resposta das estruturas a excitações dinâmicas de forma mais precisa e menos conservativa que a presentemente utilizada na aplicação dos regulamentos em vigor.

Para dimensionar uma secção de betão armado é necessário quantificar um conjunto de *n* variáveis como o esforço normal, momentos fletores ou deslocamentos. Alternativamente a admitir a simultaneidade dos valores máximos dessas variáveis, calculados através de uma típica análise modal (recorrendo a espectros de resposta e um critério de combinação modal), o método estudado avalia a correlação entre variáveis, calculando a sua superfície de interação num espaço coordenado de *n*-dimensões.

Para além de ser um problema teórico interessante, tem também claras aplicações no dimensionamento de pilares sujeitos a flexão composta ou desviada, podendo levar a reduções significativas das taxas de armadura, permitindo uma economia de materiais sem comprometer a segurança estrutural.

Keywords: Seismic analysis / Modal combination methods / Stress-resultant interaction

Palavras-chave: Análise modal / Métodos de combinação modal / Interação de esforços

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1 Introduction

The design of a reinforced concrete section relies on the evaluation of a set of variables, such as axial forces, bending moments or displacements. These can be estimated by the response spectra method associated with a combination method for all the necessary individual linear analyses: for the n relevant modes and the $k = 3$ possible seismic directions. The most common combination methods are the ABS, the SRSS and the CQC [1].

The response spectra method provides the peak value for each of the variables. However, when more than one quantity is necessary for design, the widely used approach is to assume their most unfavorable combination. This can be overly conservative as the extreme values do not occur all at the same time. The underlying idea is to properly consider the interactions between the stress-resultants in all the n x k structural analysis. This method can produce a surface in a x-dimension space that reproduces the interactions between the x variables [2].

Afterwards this surface can be superimposed to the capacity curve of that section to optimize its design. It must be remarked that the critical combination of a set of values on the interaction surface cannot be determined without the knowledge of the resistant capacity surface. In fact, the critical combination does not necessarily include the maximum value of any of the variables, as it is the one closest to the capacity surface. Gupta [3] detailed the nature of the interaction surfaces constructed for a chosen set of design variables. It is proven that for 2 variables, as N and M in a column section, the result is an ellipse. For $n > 2$ this entity is a hyper-ellipsoid in a n-coordinate space where each point represents a set of simultaneous seismic response values. As the structures are also subjected to static loads the center of these elliptical envelopes must be shifted to include this effect. Finally, the interaction surface is completely defined and can be used for design. Being completely inscribed in a resistant capacity surface guarantees the section safety.

This paper intends to explore the construction of interaction envelopes, using not only the method presented in [2] but also new ones that allow the display of the envelopes built with different mode combinations.

2 Theory

Without loss of generality let us consider just two variables (x_1, x_2), for example (M, N), whose maximum responses are calculated by the response spectra method and then combined. The two final values of these variables can be written as a vector $\boldsymbol{x_f} = [X_{f1}, X_{f2}]^T$ and represented in a coordinate space. The classical envelope we would get is a rectangle constructed from the intersections of the straight lines defined by: $x_i = X_{fi}$ and $x_i = - X_{fi}$. As not all the points inside this rectangle reproduce a feasible response of the structure it should be reduced, see Figure 1.

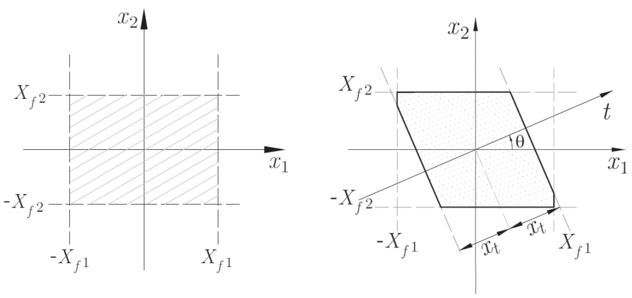


Figure 1 Rectangular and reduced envelope

The starting point for this study has to be the analysis of each mode where there is an unequivocal relation between the stress-resultants. The information of each mode, i, is organized in a vector $\boldsymbol{x_i} = [x_{i1}, x_{i2}]^T$. We also define the counterclockwise angle θ in the same coordinate space (x_1 - x_2) and its vectorial representation $\boldsymbol{t}$ = [cosθ, sinθ]^T. Considering the projection of $\boldsymbol{x}$ on any vector $\boldsymbol{t}$ is introduced a distance x_t. For one mode x_{ti} and its square value are simply defined as:

$$x_{ti} = \boldsymbol{t}^T \boldsymbol{x_i} \quad (1)$$

$$x_{ti}^{\ 2} = \boldsymbol{t}^T \boldsymbol{x_i} \boldsymbol{t}^T \boldsymbol{x_i} = \boldsymbol{t}^T \boldsymbol{x_i} \boldsymbol{x_i}^T \boldsymbol{t} \quad (2)$$

Extending this to all the n relevant modes and adopting the SRSS combination one is led to:

$$x_t^{\ 2} = \sum_{i=1}^{n} x_{ti}^{\ 2} = \sum_{i=1}^{n} \boldsymbol{t}^T \boldsymbol{x_i} \boldsymbol{x_i}^T \boldsymbol{t} = t^T \sum_{i=1}^{n} \left(\boldsymbol{x_i} \boldsymbol{x_i}^T \right) t = \boldsymbol{t}^T \boldsymbol{X} \boldsymbol{t} \quad (3)$$

Here is introduced the interaction matrix, X, which should to be adjusted to each combination method. For the SRSS combination, the definitions above direct us to:

$$\boldsymbol{X} = \sum_{i=1}^{n} \boldsymbol{x_i} \boldsymbol{x_i}^T = \begin{bmatrix} \sum_{i=1}^{n} x_{i1}^{\ 2} & \sum_{i=1}^{n} x_{i1} x_{i2} \\ \sum_{i=1}^{n} x_{i1} x_{i2} & \sum_{i=1}^{n} x_{i2}^{\ 2} \end{bmatrix} \quad (4)$$

On the other hand, for the CQC combination the definition of $x_t^{\ 2}$ should be adapted to:

$$x_t^{\ 2} = \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_{ij} x_{ti} x_{tj} \quad (5)$$

causing the matrix $\boldsymbol{X}$ to be rewritten as:

$$\boldsymbol{X} = \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_{ij} \boldsymbol{x_i} \boldsymbol{x_j}^T \ \ with \ \ \mu_{ij} = \frac{8\xi^2 (1+r) r^{(3/2)}}{\left(1-r^2\right)^2 + 4\xi^2 r (1+r)^2} \ \ and \ \ r = \frac{p_j}{p_i} \quad (6)$$

The value x_t is the maximum distance any point of the envelope can take along the direction $\boldsymbol{t}$. This is why we can add two new straight lines to the set of lines that define the boundary of the interaction envelope. These new lines are perpendicular to $\boldsymbol{t}$ at a distance of x_t from the origin and will reduce the size of the original rectangular envelope, see Figure 1.

2.1 Intersection method

Continuing the previous derivation, it should be noted that the point defined by $\boldsymbol{p} = x_t\, \boldsymbol{t}$ is not a point of the interaction envelope. In fact, the point of the envelope can be any one belonging to the line perpendicular to $\boldsymbol{t}$ that contains $\boldsymbol{p}$, see the lines R_A and R_B in Figure 2.

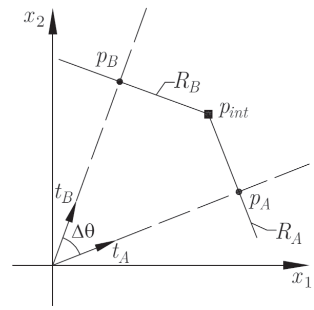


Figure 2 Intersection method: p_{int}

Taking two values of θ sufficiently close and their x_t, the envelope point is the intersection of the two perpendicular lines. Defining Point A as: $\boldsymbol{p_A} = x_{tA} \boldsymbol{t_A}$, with $\boldsymbol{t_A}$ associated with a given θ_A and x_{tA} given by Equation (5). The perpendicular line is: $R_A = x_{tA} \boldsymbol{t_A} + k_A \boldsymbol{n_A}$, where k_A is any real number and $\boldsymbol{n_A}$ as a realization of $\boldsymbol{n}$ = [- sinθ, cosθ]^T. Doing the same for Point B and matching the two perpendicular lines, produces the desired point, p_{int}. By applying the same procedure to a range of θ values contained in [0 - 2π], the shape of the envelope becomes apparent. The method developed, here called "intersection method", is directly applicable to the CQC and SRSS combinations by altering the matrix X. Moreover, it is possible to adapt it for the ABS combination by changing the x_t definition:

$$x_t = \sum_{i=1}^{n} \left| \boldsymbol{t}^T \boldsymbol{x_i} \right| \quad (7)$$

Each graphic on Figure 3 displays a typical resultant envelope for the chosen combination and also the previous envelopes were drawn in gray to ease their comparison (315 points were used, which corresponds to Δθ = 0.02 rad). It is apparent that the resultant shape for the SRSS combination is elliptical with a significant correlation between the two variables, meaning that one of the semi-axis is significantly larger than the other.

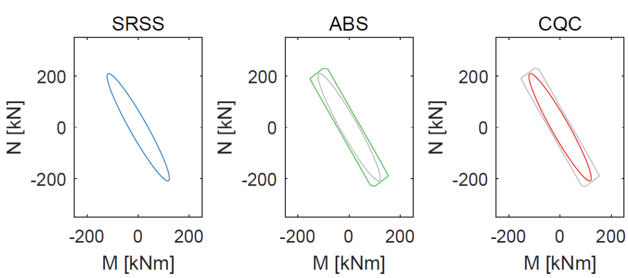


Figure 3 Intersection method: SRSS, ABS, CQC

The ABS envelope is a convex polygon exhibiting the same correlation which encloses SRSS envelope, showing it behaves as an upper bound to the other envelopes. It can be depicted as a group of pairs of parallel lines, each one introduced by one mode and whose length translates the mode relevance to the behavior of the structure. Even though the envelope could be constructed this simplified way, it is more systematic to span the θ angles, calculate its x_t and use the intersection method. Lastly, in this example the CQC envelope is so close to the SRSS solution that they overlap, meaning the modes have separated frequencies.

2.2 Resistant interaction surfaces

This solution will be obtained numerically and so the cross-section is subdivided in $n_c = 100$ rectangular divisions with concrete properties and $n_s = 16$ circular ones in the contour, simulating the steel bars. The numbering of the divisions and the sign convention are illustrated in Figure 4.

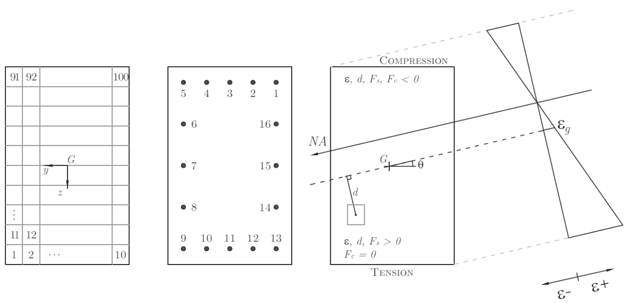


Figure 4 Discretization and sign convention

The neutral axis will be defined by two variables: the counterclockwise angle θ and ε_g, the strain at the origin, G, the geometric center of the cross-section. Since the representation of the interaction curves of N, M_2 and M_3 is a 3D surface, the intersection of this surface with horizontal planes will be drawn. These horizontal planes correspond to given values of the axial force, $N = Np$.

The angle θ will vary in 360º and for each value, will be determined the ε_g that produces the desired axial force of the horizontal plane, Np, using a Newton method. It consists in determining the maximum curvature that can be applied to the section, evaluating the strain of each concrete and steel division and calculating the stress-resultants, N, M_2 and M_3. The calculated value of the axial force, N_{calc}, will be used to adjust the initial ε_g value of the following iteration, until it matches the desired Np value.

The resultant surface, for a particular choice of steel bars, is presented in Figure 5. This figure was made using 37 iterations of θ, in each plane of $N = Np$ to complete a quarter of the total capacity surface. The maximum number of iterations to approximate the value of ε_g was 32.

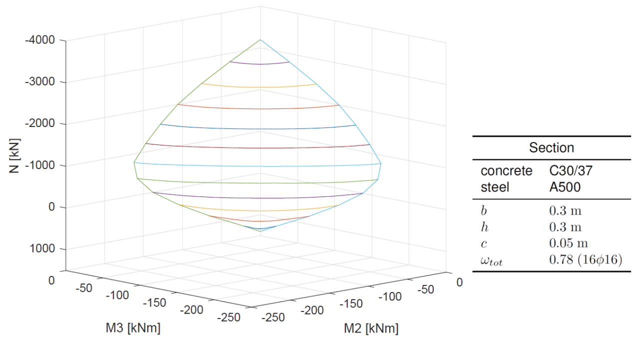


Figure 5 Interaction curves in 3D

2.3 Safety verification

To verify a section safety in a systematic way, two vectors of points that describe the resistant and action curves in polar coordinates (r, θ) were created, for each given value of N. For the action envelope it is done by inverting the interaction matrix:

$$X^{-1} = \left[\begin{array}{c|c} C_{MM} & C_{MN} \\ \hline C_{MN}^{T} & C_{NN} \end{array}\right] \tag{8}$$

With these smaller matrices we can write the equation of the ellipses that result of the intersection of the ellipsoid with horizontal planes at given values of N:

$$\left(\boldsymbol{m}^T - \boldsymbol{m_0}^T\right)\frac{\boldsymbol{C_{MM}}}{R_0^{\,2}}\left(\boldsymbol{m} - \boldsymbol{m_0}\right) = 1$$

$$\text{with} \quad \boldsymbol{m} = \left[M_2, M_3\right]^T \tag{9}$$

$$\boldsymbol{m_0} = -\boldsymbol{C_{MM}}^{-1}\,\boldsymbol{C_{MN}}N$$

$$R_0^{\,2} = 1 - NC_{NN}N + \boldsymbol{m_0}^T\boldsymbol{C_{MM}}\boldsymbol{m_0}$$

The resistant surface is also defined by an assembly of intersections with the same horizontal planes. We divide the plane into sectors centered in each point of the resistant vector and analyze if any of the action vector points are within that section. If so we compare all the action point radii in the sector with the radius of the resistant point. Given the case that in all sectors, all action point radii are smaller than the resistant point radius, we can ensure the safety of the cross-section, Figure 6.

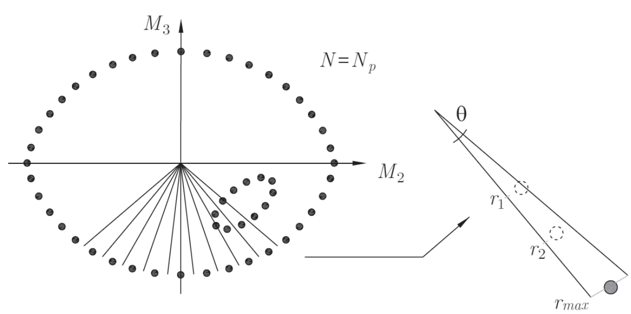


Figure 6 Safety verification mechanism

Additionally, this verification can be done inside a cycle that increments the value of the total percentage of reinforcement, w_{tot}, allowing to find the optimized solution for the cross-section design. Note the increments do not need to be fixed and can be associated with a specific choice of commercial diameters for the steel bars.

3 Implementation

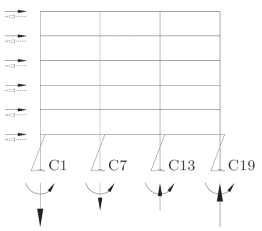
All the presented examples assume the same seismic action whose main parameters are in Table 1. [4]

Table 1 Seismic definition

Earthquake	Zone	Ground	γI	q
Type 1	1.3	Type B	1	2.5

3.1 2D Example

This example will address the design of the base section of the columns of a simple 2D structure, see Figure 7. To resist the overturning moment of the seismic action (schematically represented as a horizontal force that can have both directions), the structure has two main mechanisms: bending moments in the fixed supports and a frame effect, materialized by axial forces with symmetrical signs in each pair of columns.



story height	span	concrete	steel
3 m	6 m	C30/37	A500
columns	beams	load	
0.3×0.3 m^2	0.3×0.6 m^2	34.5 kN/m	

Figure 7 2D example: Structure description

As it can be seen in Figure 8 there is a substantial difference in the static values of *N* between interior and exterior columns. Additionally, the exterior ones have larger static *M* that slightly deviate the center of the envelopes. The seismic action produces a symmetric outcome between the left and right columns: the base sections have equal *M* combined with symmetric *N* to create a binary. This phenomenon is predominant in the outside columns which can draw larger values of *N*. The critical column is *C*13, requiring $w_{tot} = 0.5$.

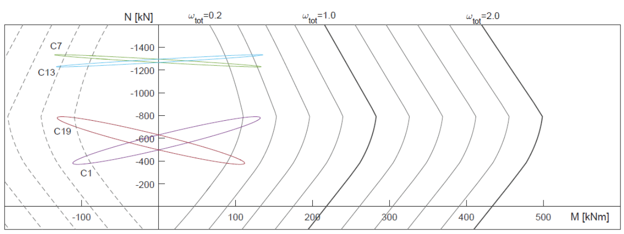


Figure 8 2D example: Action and resistant interaction curves

Additionally, we should analyze column *C*1, isolated in Figure 9. In a dashed blue line is indicated the rectangular envelope commonly used for design. With this criteria would be necessary to design the section for $w_{tot} = 0.4$ ($A_s = 17$ cm^2). Conversely, using the interaction envelope $w_{tot} = 0.3$ would suffice. This corresponds to $A_s = 12$ cm^2 that can be materialized with 12ϕ12, allowing a saving of 25%.

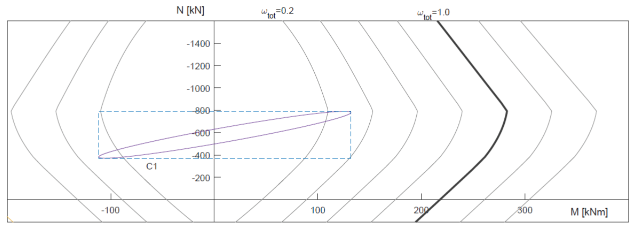
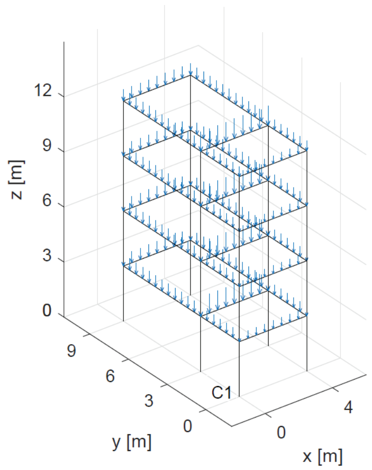


Figure 9 2D example: Comparison of envelopes for column *C*1

3.2 3D Example

The subject of the next study is a 3D structure that has an asymmetrical disposition of the columns, intended to correlate one earthquake direction to both bending moments, M_2 and M_3. The same material and geometrical properties of the 2D structure were used, plus the ones indicated in Figure 10.



concrete	C30/37
steel	A500
columns	0.3×0.3 m^2
beams	0.3×0.6 m^2
story height	3 m
span x	4 m
1st span y	3 m
2nd span y	6 m

Figure 10 3D example: Structure description

To ease the visualization of the interaction ellipsoid, constructed for the base section of the column *C*1 with coordinates ($x = 0$; $y = 0$), its projections in the three coordinate planes are presented in Figure 11. The individual responses for the earthquakes acting along the *x* and *y* directions were also presented.

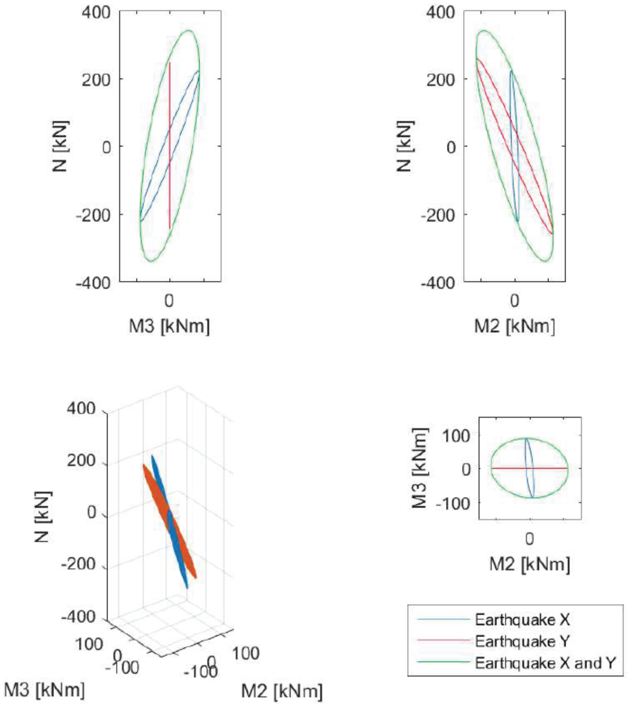


Figure 11 3D example: Ellipsoid projections (column $C1$)

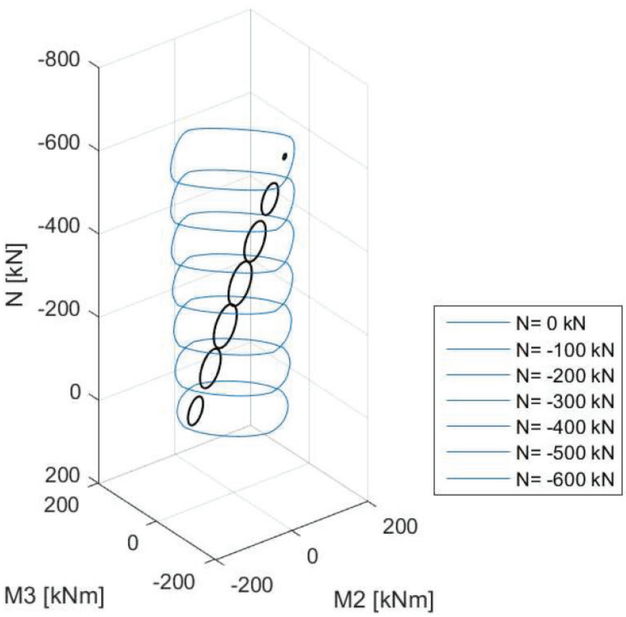


Figure 12 3D example: Action and resistance curves for given values of N (column $C1$)

Additionally, the x global axis is aligned with the e_2, in the elements referential, and the y with e_3. As it can be seen, the direction X produces N and both moments, as the direction Y only produces N and M_2. Again this can be explained by the symmetry in the disposition of columns in one direction but not in the other. FFor design purposes, the static stress-resultants have to be added, which will change the position of the ellipsoid center. With the correct placement, the action interaction ellipsoid can be compared with the resistant curves, using the safety verification method described previously.

Based on the results it can be concluded that the optimal total percentage of reinforcement is w_{tot} = 0.7. This corresponds to A_s = 29 cm^2 that may be materialized with $16\phi16$. Visually, it is clear in Figure 12 that this section verifies the safety criteria as the action curves are within the resistance curves for each value of the axial force, N, chosen to plot the curves.

4 Conclusions

The present paper was elaborated with the main goal of exploring the construction and application of interaction surfaces to the design of structures subjected to seismic actions.

The interaction surfaces replace the common practice of assuming that the multiple stress-resultants maximum values calculated with a response spectrum method can be simultaneous. This can lead to an over-design of a structure since the most unfavorable combinations of all the stress-resultants relevant for the design of each section are picked. The interaction surfaces refine the data used for design by evaluating the correlations between those stress-resultants in each vibration mode and combining them properly. It also presents a geometric representation of points whose coordinates are sets of stress-resultant values (for example: M_2, M_3 and N) that can occur together in a given section.

To support the use of these interaction surfaces we remark that it resorts to the information typically calculated in seismic analysis, based on modal combinations and response spectra. The increase of computational effort to process that information can be compensated by the material savings it allows in the design phase, by being less conservative to describe the effects of the seismic action. Such savings are described by specialized literature and reconfirmed in this paper.

Using the intersection method to construct the interaction envelopes allowed the comparison of the three modal combinations (SRSS, CQC and ABS), not just when applied to one variable but to combinations of two variables (such as M - N). As a result, graphical representations of these common combinations in 2D were produced.

The safety verification of a reinforced concrete cross-section was implemented by defining both the action and resistant interaction surfaces as an assembly of intersections with the same horizontal planes (which represent given values of axial force, *N*). Additionally, it was developed a procedure to automatically calculate the necessary steel ratio of a list of desired sections: the top and base sections on columns.

Furthermore, the principle of establishing correlations of variables in each individual mode and constructing a final interaction surface can be applied in different contexts such as the design of combined footings.

These examples are detailed in the companion dissertation [5] as they are not commonly addressed in the specialized literature.

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