Soil-pile-structure seismic interaction considering the non-linear behaviour of soil and reinforced concrete

Interação sísmica solo-estaca-estrutura considerando o comportamento não linear do solo e do betão armado

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Abstract

The aim of this article is to investigate the soil-pile-structure interaction during earthquake loading. The non-linear behaviour of the materials, soil and reinforced concrete is taken into account in the analysis. Both kinematic soil-pile interaction and inertial pile-structure interaction are studied separately and together, as well as the design of reinforced concrete pile subjected to prescribed displacement field. The soil-pile interaction was modelled using CINEMAT computational program. This software integrates the Beam on a Dynamic Winkler Foundation model (BDWF), a one-dimensional seismic wave propagation model and the linear equivalent method to account for the soil's non-linear behaviour. The non-linear behaviour of reinforced concrete pile subjected to prescribed displacements is modelled with the PIER computational program. The effect of the kinematic interaction is evaluated for particular scenarios as well as the seismic global response of a soil-pile-structure system in an alluvium formation. Seismic interaction effects are analysed, and some design considerations are presented.

Resumo

Neste artigo é estudado o comportamento de estacas de betão armado sob ações sísmicas, considerando o comportamento não linear dos materiais, solo e betão armado. Estudam-se os efeitos de interação cinemática e inercial, em separado e como fenómeno conjunto. Também se analisa o dimensionamento estrutural da estaca de betão armando sob um campo de deslocamentos imposto. A modelação do comportamento não linear do solo é realizada através do programa CINEMAT, que resulta da combinação do modelo Beam on a Dynamic Winkler Foundation (BDWF) e de um modelo de propagação unidimensional das ondas de corte sísmicas. O programa PIER é utilizado para avaliar a capacidade de deformação da estaca de betão armado através de análises fisicamente não lineares. É estudada a influência da interação cinemática em casos particulares e a resposta global da interação sísmica de um sistema solo-estaca-estrutura atravessando uma formação aluvionar. Analisam-se os efeitos de interação solo--estaca-estrutura e apresentam-se algumas recomendações para o dimensionamento de estacas de betão armado.

Keywords: Seismic soil-pile-structure interaction / BDWF (beam on dynamic winkler foundation) / Non-linear behaviour of soil / Non-linear behaviour of reinforced concrete

Palavras-chave: Interação sísmica solo-estaca-estrutura / Modelo BDWF / / Comportamento não linear do solo / Comportamento não linear do betão armado

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PISCO, G. [*et al.*] – Soil-pile-structure seismic interaction considering the non-linear behaviour of soil and reinforced concrete. **Revista Portuguesa de Engenharia de Estruturas**. Ed. LNEC. Série III. n.º 11. ISSN 2183-8488. (novembro 2019) 17-26.

1 Introduction

Earthquakes are among the most destructive forces that occur in nature. The earthquake in Japan's Kobe region in 1995, caused extensive damage, and ever since, several studies have been conducted to better understand all the phenomenon involved. One of the most important is the soil-pile-structure interaction.

Despite all the scientific work produced and all the studies carried, it has yet failed to reach a consensus for a globally accepted methodology in pile design subjected to seismic actions. Most studies conducted considered the materials' behaviour as linear elastic which is inaccurate since they assume a non-linear behaviour.

The aim of this paper is to study the seismic soil-pile-structure interaction, disregarding the effect of soil liquefaction, by analysing the inertial forces and imposed displacements given by the soil response. This phenomenon is generally known as inertial and kinematic interaction. The soil-pile-structure interaction is modelled based on the BDWF model ("Beam on Dynamic Winkler Foundation") and the soil's non-linear behaviour is modelled through the linear equivalent method. The non-linear behaviour of reinforced concrete is considered by implementing adequate constitutive laws.

As studied in previous work [1], the structural pile's response is analysed by kinematic variables as opposed to static ones by comparing the seismic induced curvatures and ultimate curvatures of the reinforced concrete sections.

2 Seismic soil-pile-structure interaction

The propagation of seismic waves through the soil causes it to vibrate, and, consequently, the vibration of a structure with a set foundation. When a structure is designed to be supported over piles, a natural interaction occurs between soil, pile and structure. This phenomenon results in additional horizontal loadings that need to be accounted, as, if not, may result in extensive damage to the piles. In the elastic domain this phenomenon can be divided into two types of loading, kinematic and inertial forces.

The kinematic forces result from the imposed displacement along the pile length due to the vertical seismic waves propagation in surrounding soil. This phenomenon is especially severe near the interface between soil layers with highly contrasting stiffness due to the high imposed curvatures.

The inertial forces result from the seismic displacements of the superstructure. These forces are proportional to the mass and acceleration of the structure and are transmitted to the foundation as concentrated horizontal forces and bending moments at the head of the pile.

2.1 BDWF model (Beam on Dynamic Winkler Foundation)

To model the interaction between soil and pile, the Beam on Dynamic Winkler Foundation model was implemented. In the BDWF model, a set of springs $k(x)$ and dampers $c(x)$ are applied through the length of the pile to replicate the effect of the soil over the pile displacement during an earthquake. By relating the free field displacements of the soil, and the above-mentioned springs and dampers, it is possible to obtain the pile's displacements and therefore to simulate the kinematic soil-pile interaction (Figure 1).

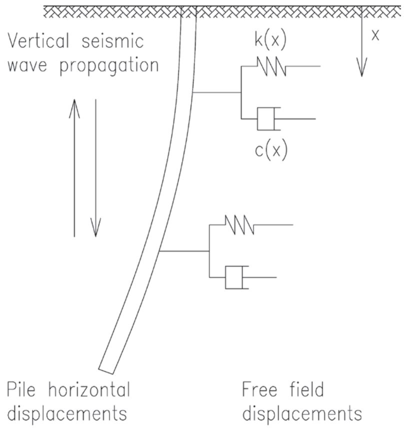

Figure 1 BDWF Model

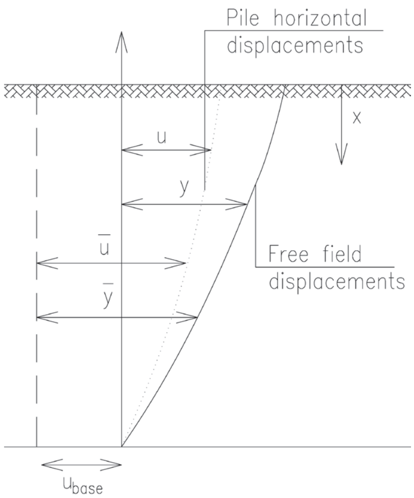

Figure 2 Flores-Berrones and Whitman Model

The model was proposed by Flores-Berrones e Whitman [2], Figure 2, and was considered for the case of a single pile in an elastic soil media with no damping. Several improvements were implemented, from which stand out the soil damping [3], the soil layering and the frequency domain analysis [4]. Santos [5] considered the non linear behaviour of soil by implementing the equivalent linear method.

In the BDWF model, the pile movement, y, is obtain by the following equations. The first one is in time domain and the second one in the frequency domain.

$$E_p I_p \frac{\partial^4 y}{\partial x^4} + \bar{m}\frac{\partial^2 \bar{y}}{\partial t^2} + c\frac{\partial(\bar{y}-\bar{u})}{\partial t} + k(\bar{y}-\bar{u}) = 0 \quad (1)$$

$$E_p I_p \frac{\partial^4 y}{\partial x^4} + (k - \bar{m}\omega^2 + ic\omega)\bar{y} - (k + ic\,\omega)\bar{u} = 0 \quad (2)$$

$E_p I_p$ is the pile's bending stiffness, $\bar{m}$ is the distributed mass, c the soil damping coefficient and k is the spring's stiffness of the model.

The soil damping can be obtained by the sum of hysteretic soil damping and radiation damping:

$$c(x) \approx c_m(x) + c_r(x) \quad (3)$$

The radiation damping coefficient can be obtained following Gazetas and Dobry's formulation ([6] and [7]). The hysteretic damping coefficient is calculated with the following expression:

$$c_m = \frac{2k(x)\xi}{\omega} \quad (4)$$

Based on [7], the spring's stiffness depends on pile head's fixity conditions and the elastic modulus of the soil:

$$k(x) \approx \delta E_s(x) \quad (5)$$

- Free head pile: $\delta = 2.1$
- Fixed head pile (no rotation): $\delta = 1.2$

The BDWF model, associated with an one-dimensional seismic wave propagation model to determine the soil displacement field u, was implemented in the code CINEMAT, [5]. This program was used in this study to determine the effect of the non-linear behaviour of the soil on the kinematic interaction between the pile and the surrounding soil.

3 Design of reinforced concrete structural elements subjected to prescribed displacements

The behaviour of reinforced concrete elements is clearly non-linear when subjected to large imposed displacements or curvatures. The usual verification of resistant capacity, based on linear elastic response and the behaviour factor, is not valid is this scenario. In the non-linear domain, static variables are not enough to characterize the $M - X$ (bending moment-curvature) values for a given section. The verification must rely on kinematic variables (deformations or curvatures) instead of static ones (stresses and forces).

In this study, the resistant capacity of a reinforced concrete section is defined by the ultimate curvature of the pile section, which is compared to the imposed curvature in order to determine the structural failure of the pile.

The non-linear response of the reinforced concrete section (concrete+steel) was modelled based on realistic constitutive laws, [8] and [9]. Additionally, it is fundamental to understand how the geometric characteristics and the materials influence the ductility of a reinforced concrete section. According to [1], the ultimate curvature of a section is function of:

- Section confinement – The ultimate concrete compressive strain, and so the ultimate curvature, is proportional to the section confinement.
- Section dimensions – The yielding and ultimate curvatures are inversely proportional to the dimension in the flexural direction.

The perpendicular one has a smaller effect on the above-mentioned characteristics.

- Material resistance – In accordance with the previous point related to section dimension, to achieve smaller areas both concrete and steel resistance should be higher in order to verify safety to the other load cases.
- Axial forces – The axial forces have a high impact in the ultimate curvature. The axial forces and the ultimate curvature have a reverse relation. This aspect is contrary for the yielding, but the impact is much lower.
- Reinforcing steel bars' design – The amount of reinforcement of a section has almost no effect in the yielding curvature. For the ultimate one, higher percentage of longitudinal reinforcement results in a decrease of the ductility.

A proper computational code was used in order to incorporate the constitutive law of each element section based on geometric and material properties, FLEXAO [1]. A second program was used in order to analyse the structural behaviour of the pile, PIER [1] for a prescribed displacement field. The imposed curvatures were determined based on the displacements and the sections characterized in FLEXAO.

4 Iterative calculation process

The material's non-linear behaviour implies a relation of dependence between impose curvature, stiffness of the soil and stiffness of the pile. In order to calculate one of the mentioned variables is necessary to determine the others, so the process is iterative.

In a simplified way, the procedure begins with the definition of the $G/G_0 - \gamma$ $\xi - \gamma$ curves and the elastic properties of the pile in the CINEMAT code. In this first phase, pile elastic displacements are calculated considering the non-linear behaviour of the soil (kinematic interaction). In the second stage, the reinforced concrete sections are defined, and the constitutive laws are calculated in the code FLEXAO. In the PIER code the displacement field is imposed to the pile. The constitutive laws of all sections of the pile are considered in the calculation in order to obtain the imposed curvatures and the new pile equivalent stiffness. In this stage the non-linear behaviour of both reinforced concrete and soil are considered. The process converges when the variation of the pile stiffness is less than 1% between consecutive iterations. The calculation process is represented in Figure 3.

5 Kinematic soil-pile seismic interaction – analysis of the behaviour of a single pile in an alluvial formation

Following [5] and [10], the study of the kinematic interaction effects on piles for several combinations of pile diameters, longitudinal and transversal reinforcement in a typical alluvial formation, a pair of new scenarios were considered in order to understand the behaviour of the pile in extreme situations regarding its ductility. [10] concluded that for typical piles diameters, seismic intensity and steel reinforcement, the imposed curvatures due to the kinematic interaction were smaller than the ultimate curvature of the section.

In this paper a first scenario was studied considering a continuous flight auger (CFA) pile without reinforcement in a specified length. This span was placed in an interval between the interface of two layers with high contrast in stiffnesses. In the second scenario, a pile with low ductility was considered. In this case, all characteristics that influence the pile ductility were chosen in order to lower it. These scenarios were established in order to understand if the kinematic interaction could lead to the pile failure in extreme situations, since in normal scenarios that seems to be not expectable.

5.1 Geotechnical profile and seismic action

The geotechnical profile considered was defined by [5]. The characteristics of each layer and $G/G_0 - \gamma$ (normalised shear modulus versus distortion), $\xi - \gamma$ (damping ratio versus distortion) curves are represented in Table 1 and Figure 4.

Table 1 Soil parameters

Layer	Thickness (m)	Behaviour	Unit weight γ (kN/m³)	Poisson ratio ν	Initial shear modulus G_0 (MPa)
1 – Fill or overconsolidated layer	5	Non-Linear	19	0.3	80
2 – Alluvial clayed layer	10	Non-Linear	17	0.5	Variable 20 to 30
3 – Miocenic layer I	5	Linear w/ ξ = 1%	22	0.3	200 ($V_s \approx$ 300m/s)
4 – Miocenic layer II	–	Rigid	–	–	–

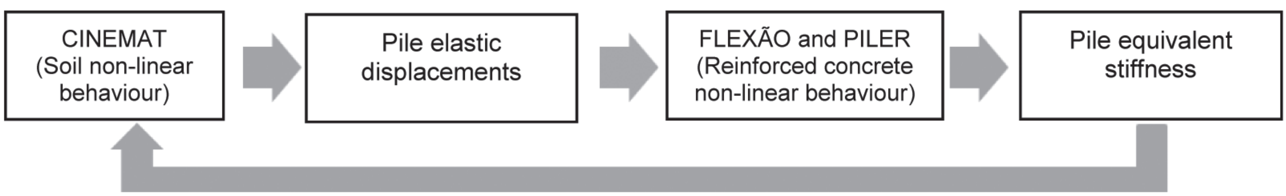

Figure 3 Iterative process

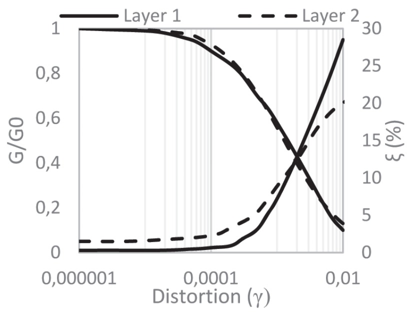

Figure 4 $G/G_0 - \gamma$ $\xi - \gamma$ curves [5]

The seismic action considered at layer 4 was the acceleration record of Kobe JMA earthquake (1995) scaled to different values of peak ground acceleration (PGA), represented in Figure 5.

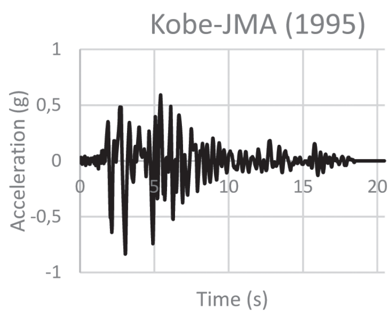

Figure 5 Kobe-JMA seismic record [5]

5.2 Distribution of pile reinforcement

As prescribed in the EC8, [10] divided the pile length in plastic zones and elastic zones. The plastic zones were placed where the imposed curvatures are higher, the pile head and the interfaces between soil layers. In the CFA pile, the first 12 m were reinforced with section S1 and the remaining part does not have any reinforcement. In Figure 6 and Table 2 are represented the different sections considered in the case of the pile with low ductility.

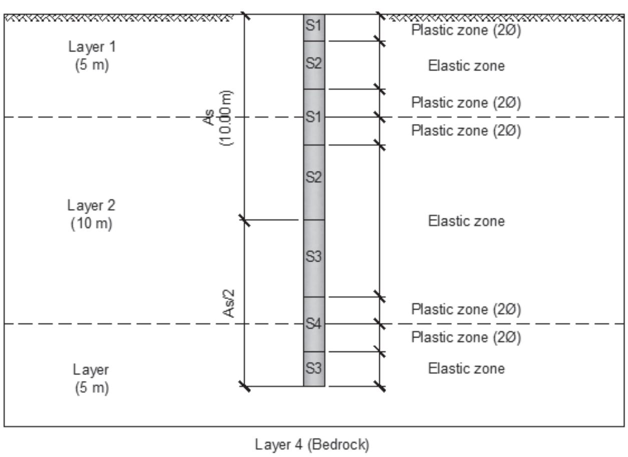

Figure 6 Pile sections

Table 2 Sections reinforcement

Section S1 and S4	Longitudinal – 18∅25
	Transversal – Plastic Zone ∅12//0.10
Section S2 and S3	Longitudinal – 18∅25
	Transversal – Elastic Zone ∅12//0.175

In Table 3 are represented the obtained values for yielding and ultimate variables for the three sections in analysis.

Table 3 Yielding and ultimate sections properties

	χ_c (‰/m)	χ_u (‰/m)	M_{ced} (KNm)	M_u (KNm)
Section S1/S4	3.66	23.28	3837	4485
Section S2/S3	3.77	16.05	3702	4164
Without reinforcement	3.29	6.98	2224	2282

5.3 Case studies

5.3.1 CFA pile with non-reinforced length

Continuous flight auger (CFA) piles is one of the most used techniques to execute pile foundations. With this type of technique, the reinforced length is usually limited to 12 m but since is a less expensive solution is commonly employed. In terms of seismic response, the mention drawback may compromise the integrity of the pile. Since is a technique often used, is important to understand how it behaves and if the ultimate curvature is achieved under seismic loading. Only the kinematic interaction was considered because, at this depth, the effects from the inertial forces are almost null. In Table 4 and Figure 7 represents the pile curvature diagrams and the obtained curvatures for the several peak ground acceleration values on bedrock.

Table 4 CFA pile case results

	$\chi_{imposed}$ (‰/m)	$\frac{\chi_{imposed}}{\chi_c}$	$\frac{\chi_{imposed}}{\chi_u}$
PGA = 0.05 g	0.28	0.09	0.04
PGA = 0.10 g	3.25	**0.99**	0.47
PGA = 0.15 g	7.61	2.31	**1.09**
PGA = 0.30 g	16.73	5.10	2.40

The results show that, for small to medium PGA values, the impose curvatures are higher than the ultimate curvature of the non-reinforced length. Another important aspect is the small difference between PGA values that correspond to the quasi-elastic behaviour, the yielding point and the collapse of the pile. For a peak ground acceleration of 0.10 g the yielding is reached and for 0.15 g the pile collapses, showing the brittle behaviour of the element. Considering these results, piles with a non reinforced length should not be considered in seismic zones.

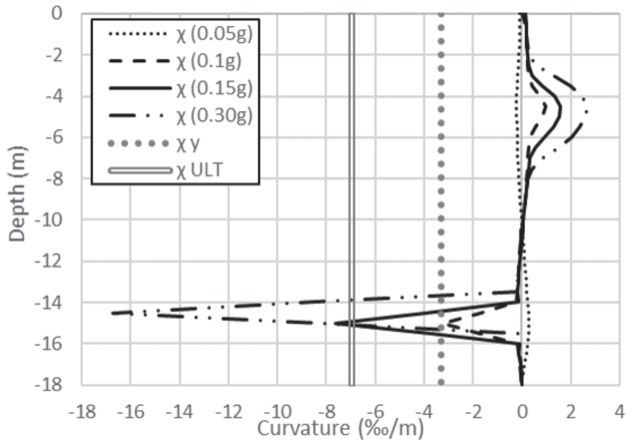

Figure 7 Curvature distribution – CFA pile

5.3.2 Pile with low ductility

In this case a pile with low ductility was simulated: large diameter (1.30 m), low percentage of confinement reinforcement, medium-high compression stress (7 MPa) and high seismic acceleration (PGA = 0.5 g). The most important calculation results in terms of curvatures are resumed in Table 5. Figure 8 represents the diagrams of curvatures of the pile for linear elastic behaviour and non-linear behaviour. For the non-linear behaviour, two curves are plotted: i) the maximum curvature at the upper interface (NL SUP) and ii) the maximum curvature at the lower interface (NL INF).

Table 5 Pile with low ductility case results

	$\chi_{imposed}$ (‰/m)	χ_y (‰/m)	χ_u (‰/m)	$\frac{\chi_{imposed}}{\chi_c}$	$\frac{\chi_{imposed}}{\chi_u}$
Section S1/S4	6.51	3.66	23.28	1.78	0.28
Section S2/S3	2.30	3.77	16.05	0.61	0.14

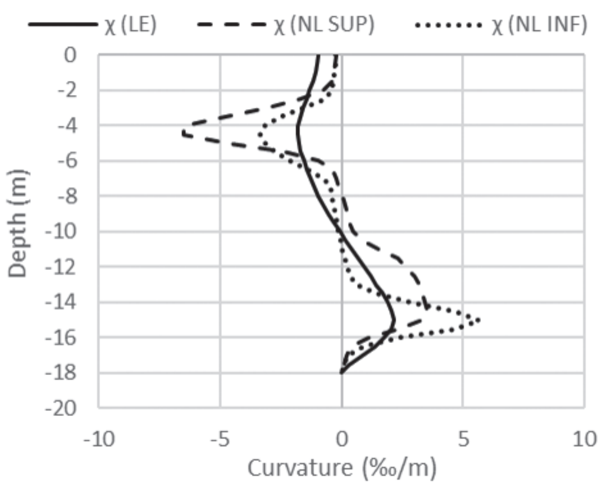

Figure 8 Curvature distribution – Pile with low ductility

For the case of a pile with low ductility is possible to concluded that, despite reaching the plastic zone, the imposed curvature is still smaller than the ultimate curvature of the pile. Even for extreme conditions, if a pile is designed for the other loads it may be subjected to kinematic effects without collapsing. However, due to the level of plastic deformation, damages and local plastic hinges are expected.

6 Validation of the BDWF model applied to the global seismic interaction soil-pile-structure phenomenon

The BDWF model, previously applied to the kinematic interaction, was generalized to consider the inertial effect of the structure in the pile. In this new model, the structure was represented by single degree of freedom oscillator with a concentrated mass at the top of a single beam element with 9 m length. To consider the structure in the model, some modifications were implemented in the BDWF model and in the CINEMAT code:

- Introduction of concentrated masses in the nodal points to simulate the structure's mass.
- Introduction of parameters to control the effect of the model springs, F_k, the soil damping, F_c, and soil mass, F_m. These three parameters range from 0 to 1. For a zero value the soil action is not considered in the nodal points and for the value of 1 the soil influence is fully activated.
- Variation of the proportionality factor, δ, between the spring model stiffness and the Young modulus of the soil. As mentioned before, this factor depends on the boundary conditions at the pile head. Since the structure is now implemented, the pile head is between free and fixed condition. This parameter is a new variable in the model.

Based on these changes, the equation (1) assumes a new form:

$$E_p I_p \frac{\partial^4 y}{\partial x^4} + F_m \times \bar{m} \frac{\partial^2 \bar{y}}{\partial t^2} + F_c \times c \frac{\partial(\bar{y} - \bar{u})}{\partial t} + F_k \times k(\bar{y} - \bar{u}) = 0 \tag{6}$$

The concentrated masses are simulated as applied nodal forces that depend on the node's acceleration.

In order to validate the newer version of the model, a single degree of freedom (s.d.f.o.) oscillator with a piled foundation in an elastic soil layer was simulated with CINEMAT and a three-dimensional finite element model (SAP software [11]). The response of the system depends on the relation between the frequencies of the soil and the structure, and the s.d.f.o. response was obtained from the theoretical solution presented by [12]. The considered model is represented in Figure 9.

The model was validated based on the next comparisons:

1) Kinematic interaction for pile with distributed mass versus concentrated mass on the nodes;
2) Single degree of freedom oscillator in both CINEMAT and 3D model;
3) Global soil-pile-structure seismic response in both CINEMAT and 3D model for different soil/structure frequency ratios.

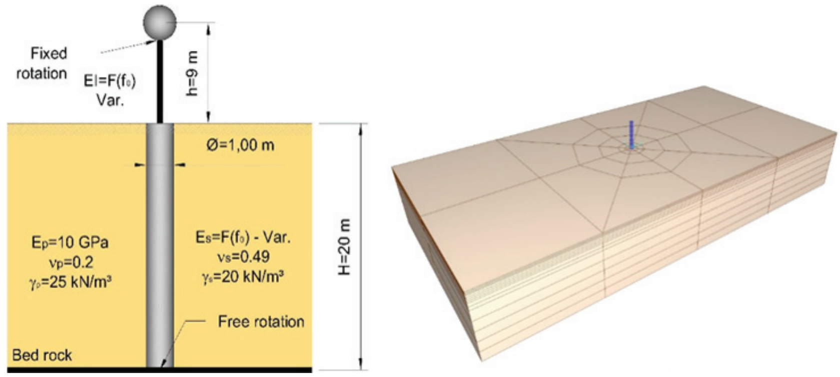

Figure 9 Model properties and finite element mesh

In the first two comparisons, both models show very good agreement. In the third comparison, considering that the behaviour of the system strongly depends on the relation between the soil and structure frequencies, three different scenarios were analysed. In the first, the fundamental frequencies of the soil and of the structure were the same. In the second, the fundamental frequency of the soil was 3 times the frequency of the structure. In the last one, the ratio was 1/3.

As expected, the results showed a high sensitivity to the coefficient δ, which depends on the pile head fixity conditions. In the global interaction, since the structure is directly modelled, the previous considerations were no longer valid. For instance, the coefficient δ depends on the fundamental frequencies and their ratio. Considering this, in all simulations the coefficient δ was chosen in order to achieve the best possible match in the results of the two models. In Figure 10 to Figure 15 are represented the transfer functions obtained for both models. The small differences observed in terms of amplitude and frequencies are related to the high sensitivity to the coefficient δ, as explained before, and with the different methods to model the system damping.

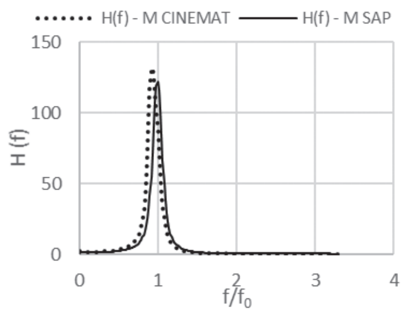

Figure 10 Bending moments transfer function for the pile head $f_{STR} = f_{SOIL}$

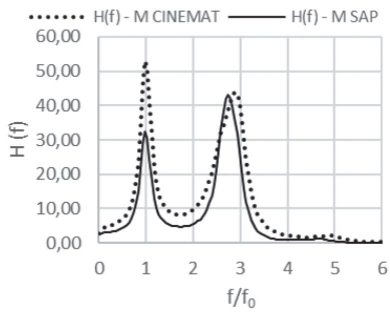

Figure 12 Bending moments transfer function for the pile head $f_{STR} = 3 \times f_{SOIL}$

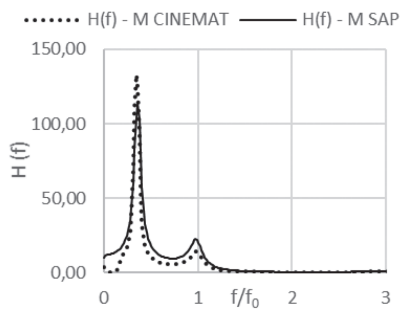

Figure 14 Bending moments transfer function for the pile head $f_{STR} = f_{SOIL}/3$

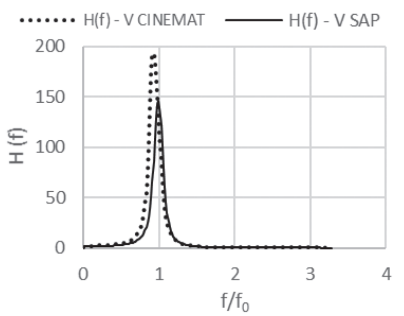

Figure 11 Shear forces transfer function for the pile head $f_{STR} = f_{SOIL}$

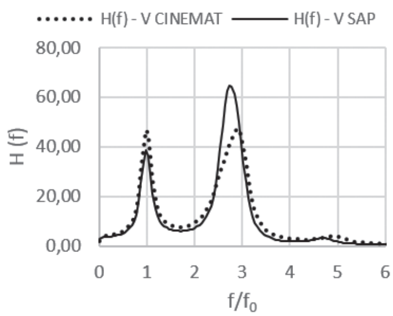

Figure 13 Shear forces transfer function for the pile head $f_{STR} = 3 \times f_{SOIL}$

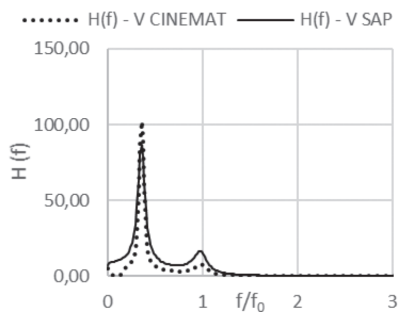

Figure 15 Shear forces transfer function for the pile head $f_{STR} = f_{SOIL}/3$

Both models provided expected results in every scenario. When the frequency of the structure is higher than the soil, both kinematic and inertial effects are in phase. When the structures have a smaller frequency both phenomena are out of phase. In the last case the inertial part governs the behaviour of the system.

In terms of δ, for the case in which the frequencies were the same or the structure's was smaller the value was about $\delta = 1.2$. When the soil fundamental frequency was higher the value that corresponded to the best match was $\delta = 2.1$.

The results confirm that the updated BDWF model can provide reliable results for the study of the seismic soil-pile-structure interaction.

7 Seismic soil-pile-structure interaction

The effect of the global seismic interaction phenomena was studied considering a pile with low ductility in a multi layered media. The geotechnical profile and seismic action were the same applied before in the kinematic interaction study. Considering the previous results, the initial soil and structure frequencies were kept the same. The structure was simulated by a single degree of freedom oscillator with 9 m high, with linear elastic behaviour. The peak ground acceleration on bedrock was 0.05 g and 0.10 g. Figure 16 presents the results obtained in terms of curvatures.

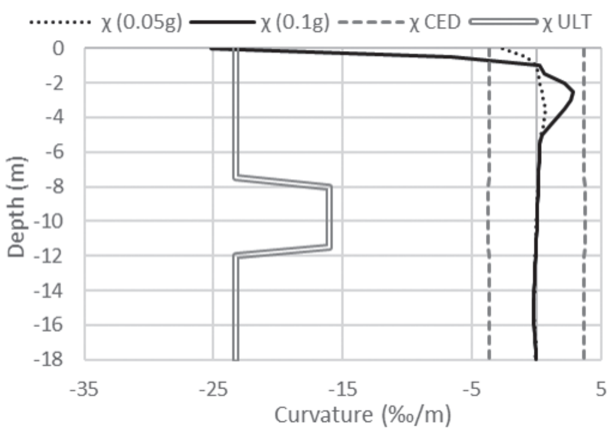

Figure 16 Curvature distribution considering soil-pile-structure interaction

It is clear from Figure 16, the pile collapses when considering the global seismic interaction phenomenon. The inertial and kinematic effects are out of phase, since there is almost no peak in the imposed curvatures near the layer's interface. This aspect may be explained by the non-linear behaviour of the soil that caused a change of the fundamental frequency. This observation shows that the non-linearity is also important in order to understand how the different parts of the interaction may occur during the earthquake. Another important observation is the sensibility of the structure to the ground acceleration. In this case study, a variation of two times in the PGA value leads to an increase in the maximum curvature of ten times, passing from a yielding situation to the collapse of the pile.

The non-linear behaviour of the soil and pile are fundamental to the correct understanding of the effects of seismic soil-pile-structure interaction.

8 Conclusions

Results from the study on kinematic interaction and global seismic soil-pile-structure interaction were presented and discussed, highlighting several important issues. The main conclusions from this study are:

Kinematic interaction

- If the pile is correctly designed for the other loads it can yield but will not collapse and the level of damage is inversely proportional to the ductility of the reinforced concrete pile.
- Concrete piles with non-reinforced lengths should not be used in medium to high seismic hazard zones.

Soil-pile-structure interaction

- The updated BDWF is accurate and can be a useful design tool for current practice. The model is quite sensitive to the parameter δ. More studies about the impact of this parameter in the global response of the system should be carried out.
- The kinematic and inertial effects when combining together can lead to the collapse of the pile. More studies should be carried out for diverse geotechnical scenarios and with other type of structures to support the conclusions of this study.

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