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Calibration of partial safety factors using FORM

Calibração de coeficientes parciais recorrendo ao método FORM

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Abstract

Partial safety factors present in codes are defined for the design of broad populations of structures. As a consequence, they do not always reflect correctly the uncertainties for specific existing structures. The possibility of adjusting them has therefore great interest when assessing an existing structure. The concept of design point connected to FORM offers a simple method to calibrate partial safety factors for individual variables. This paper discusses the overall methodology and develops a set of analytical expressions for the probabilistic models more common in structural reliability.

As it will be seen the partial factor for a particular variable can be adjusted once defined: (1) probabilistic model used to describe the uncertainty in that variable; (2) coefficient of variation of the variable; (3) fractile implicit on the characteristic value used to quantify the design value; (4) importance of the variable in the limit state under consideration (measured by the respective sensitivity factor), and (5) target reliability index.

The choice of the probabilistic model influences significantly the partial factors and this influence rises as the coefficient of variation increases. As a consequence, more attention must be paid when choosing a probabilistic model for a variable with high coefficient of variation.

Resumo

Os coeficientes parciais de segurança especificados nos regulamentos estão calibrados para o dimensionamento de populações de estruturas relativamente vastas. Consequentemente, tais coeficientes nem sempre refletem corretamente os níveis de incerteza envolvidos em estruturas específicas existentes. A possibilidade de ajustá-los tem, por isso, grande interesse na avaliação de estruturas existentes. O conceito de ponto de dimensionamento FORM oferece um método simples para calibrar coeficientes parciais de segurança relativos a variáveis individuais. Este artigo discute o método geral e desenvolve um conjunto de expressões analíticas para os modelos probabilísticos mais comuns em fiabilidade estrutural.

Como veremos, o coeficiente parcial de segurança de uma variável específica pode ser ajustado uma vez definidos: (1) modelo probabilístico usado para descrever a incerteza nessa variável; (2) coeficiente de variação da variável; (3) quantil implícito no valor característico utilizado para quantificar o valor de dimensionamento; (4) importância da variável no estado limite em consideração (medida pelo respetivo coeficiente de sensibilidade) e (5) índice de fiabilidade alvo.

A escolha do modelo probabilístico influencia significativamente os coeficientes parciais e essa influência é tanto mais significativa quanto maior for o coeficiente de variação da variável. Consequentemente, mais atenção deve ser dada ao escolher um modelo probabilístico para uma variável com alto coeficiente de variação.

Keywords: Structural safety / Partial safety factors / Calibration / FORM / / Sensitivity factors / Existing structures

Palavras-chave: Segurança estrutural / Coeficientes parciais de segurança / / Calibração / FORM / Coeficiente de sensibilidade / Estruturas existentes

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1 Introduction

Calibration of partial safety factors is of interest not only to code writers but also to designers of structures and mainly those involved in the assessment of existing structures. In fact, the partial factors specified in codes for new structures are applied to vast populations of structures and might not reflect correctly the level of uncertainty for a particular existing structure [1]. If relevant statistical information regarding a given basic variable is available during the assessment of an existing structure, the possibility of adjusting the partial factor of that particular variable is of great interest, for that information can lead to a decrease of uncertainty and a possible reduction in the partial factor.

The possibility of adjusting the partial factors when assessing an existing structure can be regarded as an intermediate level between the method of partial factors, with fixed values as specified in codes for new structures, and a full probabilistic analysis, which requires probabilistic information for all basic variables involved and is more demanding from the practical point of view.

According to EN 1990 [2], the partial safety factors specified in Eurocodes were calibrated mainly based on a long experience of building tradition (clause C3 (2)). The same clause mentions that partial factors might also be calibrated using probabilistic tools. A general method to calibrate partial factors through probabilistic tools is described in [3] and in ISO 2394 [4]. This method, however, is of interest mainly to code writers since its use is not simple for practical applications. A simpler approach, based on First Order Reliability Method (FORM), has gained the attention of several researchers due to its simplicity and has strong application in the assessment of existing structures domain [5].

A recent recommendation published by fib [6] constitutes an important step to bring this approach to the practising engineers. The present paper gives a contribution on this subject, by discussing the overall methodology and by developing formulas for probabilistic models common in structural reliability but not covered by the above fib recommendation, namely Fréchet and Weibull distributions.

2 Calibration of partial factors using FORM – General formulation

Consider a limit state function $M = g(X_1, \ldots, X_n)$, whose safety margin M depends on n basic variables, X_i. The function g is defined so that $M < 0$ represents failed states, $M = 0$ limit states and $M > 0$ unfailed states [4]. According to the partial factor format, the safety to that limit state is considered acceptable if:

$$g(X_{1d,\ldots,}X_{nd}) \geq 0 \tag{1}$$

where each random basic variable X_i was substituted by its design value, X_{id}. According to FORM, representing the cumulative distribution function of X_i by $F_{Xi}(x)$, the design value X_{id} is given by[1]:

1 The point $(X_{1d},\ldots,X_{nd})$, called design point, is the point belonging to the surface $g = 0$ with the highest probability density, being therefore the point where a possible rupture is most likely to occur.

$$X_{id} = F_{X_i}^{-1}\left(\Phi(-\alpha_i\beta)\right) \quad (2)$$

where $\Phi()$ is the standard Normal cumulative distribution, α_i is the FORM sensitivity factor of X_i for the limit state under consideration and β is the reliability index for that limit state.

The sensitivity factor α_i measures the contribution of the uncertainty of the variable X_i to the reliability β [7]. For a given limit state $M = g(X_1,...,X_n)$, composed by n basic variables, the α-factors have the following properties:

$$-1 \leq \alpha_i \leq 1 \quad (3)$$

$$\sum_{i=1}^{n} \alpha_i^2 = 1 \quad (4)$$

For a basic variable X representing an action, its design value is given by $X_d = \gamma_f X_k$, where γ_f is the partial factor and X_k the characteristic value. Thus, a general expression for γ_f is:

$$\gamma_f = \frac{X_d}{X_k} = \frac{F_X^{-1}\left(\Phi(-\alpha \times \beta)\right)}{X_k} \quad (5)$$

For a basic variable X representing a resistance, the design value is given by $X_d = X_k/\gamma_m$, where X_k is the characteristic value of X and γ_m the partial factor of X. Thus, γ_m can be expressed by:

$$\gamma_m = \frac{X_k}{X_d} = \frac{X_k}{F_X^{-1}\left(\Phi(-\alpha \times \beta)\right)} \quad (6)$$

The characteristic value X_k in Equations (5) and (6) corresponds in general to a fractile p of the corresponding distribution functions F_X (e.g. $p = 0.95$ for actions and $p = 0.05$ for resistances). In this case, X_k is given by:

$$X_k = F_X^{-1}(p) \quad (7)$$

Note that γ_f and γ_m depend on the fractile p associated to the characteristic value used to compute the design value. In the case of actions, increasing p causes a decrease in γ_f, and in the case of resistances, increasing p causes an increase in γ_m.

The value of the reliability index β in Equations (5) and (6) can be regarded as the target value β_T, as specified in codes, which depends on the consequences of failure and on the marginal cost of safety [8]. Eurocode 0 [2] defines three Reliability Classes (RC1, RC2 and RC3), corresponding to low, medium and high consequences. The target values of the reliability for each Reliability Class are reproduced in Table 1. As shown, the recommended target reliability indexes are presented for two reference periods: 1 and 50 years, which correspond to the same level of reliability [9]. The column "50 years" should be interpreted as the reliability for the design working life (or residual working life, in the case of existing structures), irrespective of its duration [10]. When the design working life is 50 years, then the annual reliability has the value indicated in column "1 year".

The target reliabilities specified in Eurocode 0 [2] are intended mainly for new structures. For existing structures, it has been recognized that values lower than these are more appropriate [11], as the marginal cost of safety is greater for existing structures. This subject, although pertinent, is outside the scope of this study, so it will not be addressed here.

Table 1 Target values of reliability index as specified in [2]

Reliability Class	Consequences	Reference period	
		1 year	50 years
RC3	High	5.2	4.3
RC2	Medium	4.7	3.8
RC1	Low	4.2	3.3

The key point of the methodology for calibrating partial factors using FORM lies in the sensitivity factors. The sensitivity factor α for a particular variable varies from limit state to limit state. However, it is possible to identify certain common values of this factor. The standard ISO 2394 [4] proposes the values shown in Table 2, which were also adopted in Eurocode 0 [2]. It should be noted that these values are generally conservative. For example, for a limit state composed by only 2 basic variables, one action and one resistance, the first is naturally the dominant action, and the second is the dominant resistance. Using then the values proposed by those standards, the sum of the squares of the α-factors for that limit state is $(-0.7)^2 + (0.8)^2 = 1.13$, which is greater than 1, showing that the sensitivity factors of –0.7 and 0.8 are conservative when used together. If there are more than two basic variables, the degree of conservatism increase.

Table 2 Standardized α-values as specified in [4]

Basic variable	α
Dominating action...	– 0.70
Remaining actions (accompanying actions)......0.40(– 0.70)......	– 0.28
Dominating resistance	0.80
Remaining resistances..........................0.40(0.80).........................	0.32

According to Clause C.7 (3) of [2], the values in Table 2 should be used only if:

$$0.16 < \frac{\sigma_E}{\sigma_R} < 7.6 \quad (8)$$

where σ_E is the standard deviation of the dominating load effect and σ_R the standard deviation of the dominating resistance. If this condition is not valid, $\alpha = \pm 1.0$ should be used for the variable with the highest standard deviation and $\alpha = \pm 0.4$ for the remaining variables. The "–" sign is to be used for variables that decrease reliability (in general loads) and the "+" sign is to be used for variables that increase reliability (in general resistances).

In the next sections, the standardized α-values and Equations (5) and (6) will be used to derive expressions for partial factors of basic variables with specific probabilistic models.

In the specific case of variable actions, Eq. (5) should be applied with care. The probability distribution of variable actions refers in general to the maxima over a reference period. Obviously, the reference period implicit in β in Eq. (5) must be the same. On the other hand, the sensitivity factor α for a given variable action depends also on the reference period, as it depends on the standard deviation of the action, which, in turn, depends on the reference period. It can be assumed that the standardized α-values recommended in [2] applies to the design working life. Then, to obtain γ_f-factors for variable actions using (5), the reference period implicit in β and in the distribution of maxima should be the design working life.

Note also that the partial factors obtained from Equations (5) and (6) reflect only the uncertainty of the variable itself, as described by the function $F_X(\cdot)$, and therefore cannot be directly compared with the partial factors specified in Eurocodes, since these also reflect the uncertainty in models present in the limit state functions. Next section deals with model uncertainties.

3 Model uncertainties

A limit state function, symbolically represented by $M = g\ (X_1,\ldots,X_n)$, describes how the safety margin M is obtained from the basic variables $X_1,\ldots,X_n$. These variables are called basic in the sense that they are not obtained from others [12]. In the definition of g, however, it is necessary to use variables other than the basic ones, that is, variables that are functions of the basic ones. These functions, or models, are basically of three types, as suggested in Figure 1:

- Action models, $Q = Q(X_1,\ldots,X_n)$, which transform basic variables into loads.
- Structural models, $E = E(Q)$, which transform loads into load effects.
- Resistance models, $R = R(X_1,\ldots,X_n)$, which transform basic variables into resistances, expressed in the same domain as the load effect E.

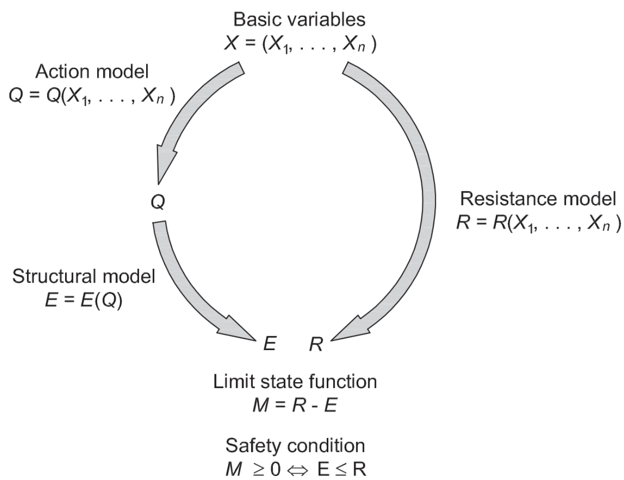

Figure 1 Models present in a typical limit state function

These models do not translate reality perfectly, due to deliberate simplifications or neglected effects, which originate new uncertainties in addition to those included in the basic variables. Although such uncertainties are clearly of the epistemic type, they should be modelled by means of random variables. These variables, usually represented by the Greek letter θ, should be added to the vector of the basic variables and are usually included in a model $Y = Y\ (X_1,\ldots,X_n)$ in one of the following formats: $Y = \theta \cdot Y\ (X_1,\ldots,X_n)$ or $Y = \theta + Y\ (X_1,\ldots,X_n)$ [13]. The first format is more common [14] and will be used in the present study.

3.1 Uncertainty in action and in structural models

Let θ_E be the random variable accounting for the uncertainties in both action models and structural models. If $E(X_1,\ldots,X_n)$ represents the load effect as given by those models, and E the true load effect, the variable θ_E is defined so that:

$$E = \theta_E \cdot E(X_1,\ldots,X_n) \tag{9}$$

Consequently, according to the partial factors method, the design value of E is given by:

$$E_d = \theta_{Ed} \cdot E(X_{1d},\ldots,X_{nd}) \tag{10}$$

where θ_{Ed} represents the design value of the variable θ_E. On the other hand, according to EN 1990 [2], E_d is given by:

$$E_d = \gamma_{Sd} \cdot E(X_{1d},\ldots,X_{nd}) \tag{11}$$

where γ_{Sd} is the partial factor associated with the uncertainty of the action and structural models. Comparing (11) to (10) it follows that the partial factor γ_{Sd} coincides with the design value θ_{Ed}. Thus, representing the CDF of θ_E by $F_{\theta E}(\cdot)$, Eq. (2) yields:

$$\gamma_{Sd} = F_{\theta_E}^{-1}\left(\Phi(-\alpha_E\beta)\right) \tag{12}$$

where α_E is the FORM sensitivity factor of θ_E.

Like other basic variables, θ_E has its own mean and standard deviation. Ideally, the mean $\mu_{\theta E}$ should be close to 1.0, but frequently is lesser than 1.0, as the models employed to compute the load effect E are often conservative. The mean $\mu_{\theta E}$ can be regarded as a measure of the *accuracy* (bias) of those models, that is, their ability to predict load effects with mean close to the real value. The standard deviation $\sigma_{\theta E}$ can be regarded as a measure of the *precision* of the same models, that is, their ability to predict load effects with little dispersion between each other. The lack of precision can be due to variables neglected in the structural analysis. For example, there may be environmental influences that may affect the response of the structure but are not normally taken into account.

The variable θ_E is in general modelled by a Lognormal distribution [13, 15].

3.2 Uncertainty in resistance models

Let θ_R be the random variable accounting for the uncertainties in the resistance model underlying a given limit state. If $R(X_1,\ldots,X_n)$ represents the resistance predicted by that model, and R the true resistance, the variable θ_R is defined so that:

$$R = \theta_R \cdot R(X_1,\ldots,X_n) \tag{13}$$

Consequently, the design value of R is given by:

$$R_d = \theta_{Rd} \cdot R(X_{1d},\ldots,X_{nd}) \tag{14}$$

where θ_{Rd} represents the design value of the variable θ_R. According to EN 1990 [2], R_d is given by:

$$R_d = \frac{1}{\gamma_{Rd}} \cdot R(X_{1d},\ldots,X_{nd}) \tag{15}$$

where γ_{Rd} is the partial factor associated with the uncertainty of the resistance model. Comparing (15) to (14) it follows that the partial factor γ_{Rd} coincides with the reciprocal of the design value θ_{Rd}. In this way, representing the CDF of θ_R by $F_{\theta R}(\cdot)$, Eq. (2) gives:

$$\gamma_{Rd} = \frac{1}{F_{\theta R}^{-1}\left(\Phi(-\alpha_R\beta)\right)} \tag{16}$$

where α_R is the FORM sensitivity factor of θ_R.

As for the variable θ_E, the mean of θ_R is a measure of the *accuracy* of the resistance model and the standard deviation a measure of its *precision*. If a resistance model is relatively accurate (small systematic errors, or small bias), the mean of θ_R will be close to 1.00. (Frequently, the mean is greater than 1.0 due to the usual degree of conservatism in the resistance models.) If, in addition, the model is relatively accurate (small random errors) the standard deviation of θ_R will be small.

The variable θ_R is in general modelled by a Lognormal distribution [13, 14].

4 Partial factors for actions

4.1 γ_f-factors

4.1.1 Actions with normal distribution

Let X be a basic variable representing an action, and assume that $X \sim N(\mu,\sigma)$. Considering the expression for the inverse of the Normal distribution, Eq. (5) leads to:

$$\gamma_f = \frac{\mu(1-\alpha\beta V)}{X_k} \tag{17}$$

where $V = \sigma/\mu$ is the coefficient of variation of the variable under consideration, α is the FORM sensitivity factor of X and β is the desired reliability index for the limit state under consideration. In the case the characteristic value X_k refer to the fractile p, Eq. (17) becomes:

$$\gamma_f = \frac{1-\alpha\beta V}{1+\Phi^{-1}(p)V} \tag{18}$$

For permanent actions due to selfweight, g, its nominal value refers frequently to the mean value (instead of a fractile). Hence, for this specific case, the partial factor is:

$$\gamma_g = 1-\alpha\beta V \tag{19}$$

4.1.2 Actions with lognormal distribution

For an action with Lognormal distribution, with mean μ and coefficient of variation V , Eq. (5) leads to:

$$\gamma_f = \frac{\frac{\mu}{\sqrt{1+V^2}}\exp\left(-\alpha\beta\sqrt{\ln(1+V^2)}\right)}{X_k} \tag{20}$$

If X_k refers to the fractile p, then $X_k = F_X^{-1}(p)$ and Eq. (20) becomes:

$$\gamma_f = \exp\left(\sqrt{\ln(1+V^2)}\left(-\alpha\beta-\Phi^{-1}(p)\right)\right) \tag{21}$$

4.1.3 Actions with Gumbel distribution

Given an action with Gumbel distribution (also known as Extreme Type I distribution of maxima) with mean μ and coefficient of variation V , Eq. (5) yields the following expression:

$$\gamma_f = \frac{\mu\left[1-0.78V\left(0.58+\ln\left(-\ln\Phi(-\alpha\beta)\right)\right)\right]}{X_k} \tag{22}$$

The Gumbel distribution is frequently recommended in modelling maxima of variable actions in a given reference period, as is the case in Eurocodes for climatic actions. Recall that, assuming that α applies to the design working life, β must also refers to the design working life (or the remaining working life for an existing structure, if defined). As a consequence, the reference period implicit in parameters μ and V in Eq. (22) must be the design working life.

In general the characteristic value of variable actions refer to the fractile p. In this case Eq. (22) becomes:

$$\gamma_f = \frac{1-0.78V\left(0.58+\ln\left(-\ln\Phi(-\alpha\beta)\right)\right)}{1-0.78V\left(0.58+\ln(-\ln p)\right)} \tag{23}$$

The fractile p is the probability that the characteristic value is not exceeded in the reference period, which, as mentioned, must be the design working life. A typical value is $p = 0.95$ [16], but this is not always the case in Eurocodes: for example, the characteristic values of climatic actions refer to the fractile 0.98 of the annual maxima. Thus, for a structure with a design working life of n years the fractile for climatic actions to be used in Eq. (23) must be $p = 0.98^n$.

As an example, consider a structure for which the wind is a dominant action ($\alpha = -0.7$), and assume a coefficient of variation of 0.13 regarding the annual maxima. Assume that the structure belongs to the Reliability Class RC2 ($\beta = 3.8$) and that the design working life is 50 years. The coefficient of variation of 0.13 must be transformed in this period of time, as follows: $V = \pi\Big/\left(\frac{\pi}{V_1}+\sqrt{6}\ln 50\right) = 0.093$. Assuming that the characteristic value is quantified according to the Eurocodes, the γ_f-factor for the wind action would be:

$$\gamma_f = \frac{1-0.78(0.093)\left(0.58-\ln\left(-\ln\Phi(0.7\times 3.8)\right)\right)}{1-0.78(0.093)\left(0.58+\ln\left(-\ln 0.98^{50}\right)\right)} = 1.42$$

4.1.4 Actions with Fréchet distribution

The Fréchet distribution, also known as Extreme Type II distribution of maxima, is also an important distribution in modelling maxima of variable actions, namely those connected with extreme meteorological phenomena [17]. Consider an action whose maxima follows a Fréchet distribution with shape parameter *k* (see Annex A for details on this distribution). The parameter *k* is directly related to the coefficient of variation. Table 3 shows values of *k* corresponding to some values of *V*.

Using once more Eq. (5) with $X_k = F_X^{-1}(p)$, the following expression for the partial factor of a variable with Fréchet distribution is obtained:

$$\gamma_f = \left(\frac{\ln p}{\ln \Phi(-\alpha\beta)} \right)^{\frac{1}{k}} \tag{24}$$

As before, for variable actions, the fractile *p*, the reliability index β and the parameter *k* must refer to the design working life.

Table 3 Values of the parameter *k* corresponding to different coefficients of variation – Fréchet distribution

V	*k*
0.05	26.41
0.10	13.62
0.15	9.37
0.20	7.26
0.25	6.01
0.30	5.18

4.1.5 Examples

Table 4 exemplifies values of γ_f-factors obtained using the expressions presented above, considering different coefficients of variation and different Reliability Classes. In all expressions the values $\alpha = -0.70$ and $p = 0.95$ were considered.

As can be seen, the Fréchet model leads to the highest partial factors, and the Normal model the lowest. This result was already expected considering the weight of the upper tails of these models, which is smaller in the Normal model and larger in the Fréchet model. It is also seen that the higher the coefficient of variation, the greater the difference in partial factors obtained in the different models. For low coefficients of variation, the differences between the 4 models is relatively small. It can thus be concluded that for high coefficients of variation the choice of the probabilistic model is more important.

It is also interesting to observe the influence of the Reliability Classes on the partial safety factors. For usual coefficients of variation (values between 0.15 and 0.20), raising a Reliability Class results in an increase of about 15% in the partial factor.

Figure 2 plots the results shown in Table 4 for the Reliability Class RC2 (β = 3.8). As it is seen, for coefficients of variation larger than 0.38, the Lognormal model becomes more conservative (in the sense that it leads to higher partial factors) than the Gumbel model.

Table 4 Partial factors for actions, γ_f ($\alpha = -0.7$, $p = 0.95$)

Model	*V*	Consequences		
		Low (β = 3.3)	Medium (β = 3.8)	High (β = 4.3)
Normal	0.05	1.03	1.05	1.06
	0.10	1.06	1.09	1.12
	0.15	1.08	1.12	1.16
	0.20	1.10	1.15	1.21
	0.25	1.12	1.18	1.24
	0.30	1.13	1.20	1.27
Lognormal	0.05	1.03	1.05	1.07
	0.10	1.07	1.11	1.15
	0.15	1.10	1.16	1.23
	0.20	1.14	1.22	1.31
	0.25	1.18	1.28	1.40
	0.30	1.22	1.35	1.49
Gumbel	0.05	1.06	1.09	1.13
	0.10	1.10	1.17	1.24
	0.15	1.14	1.23	1.34
	0.20	1.18	1.29	1.42
	0.25	1.21	1.34	1.49
	0.30	1.24	1.39	1.55
Fréchet	0.05	1.06	1.10	1.15
	0.10	1.12	1.21	1.31
	0.15	1.18	1.32	1.48
	0.20	1.24	1.43	1.66
	0.25	1.30	1.53	1.84
	0.30	1.36	1.64	2.03

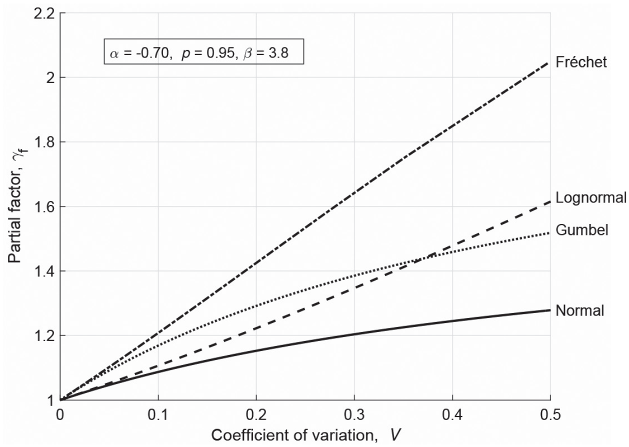

Figure 2 γ_f-factors as a function for different probabilistic models.

4.2 γ_{Sd}-factors

4.2.1 General expression

The variable θ_E is in general modelled by a Lognormal distribution [13, 15]. Thus, using the expression for the inverse of Lognormal model, Eq. (12) yields:

$$\gamma_{Sd} = \frac{\mu_{\theta E}}{\sqrt{1+V_{\theta E}^2}} \cdot \exp\left(-\alpha_E\, \beta \sqrt{\ln(1+V_{\theta E}^2)}\right) \qquad (25)$$

where $\mu_{\theta E}$ and $V_{\theta E}$ are the mean and coefficient of variation of the variable θ_E. If $V_{\theta E} \leq 0.20$, Eq. (25) can be approximated by:

$$\gamma_{Sd} \approx \mu_{\theta E} \cdot \exp(-\alpha_E\, \beta V_{\theta E}) \qquad (26)$$

The mean $\mu_{\theta E}$ and the coefficient of variation $V_{\theta E}$ reflect the accuracy and precision of the models used and should therefore be chosen carefully on a case-by-case basis.

It is possible to find in literature recommendations concerning statistical parameters of θ_E. Table 5 shows the JCSS recommendations [13]. The Danish guide [18] recommends $V_{\theta E}$ = 0.04 for structural models with good accuracy, $V_{\theta E}$ = 0.06 for models with normal precision and $V_{\theta E}$ = 0.09 for models with low precision.

Table 5 Probabilistic models for the variable θ_E, as recommended by JCSS [13]

Type of structural model	Response	Distribution	$\mu_{\theta E}$	$V_{\theta E}$
Frame models	Moments	Lognormal	1.00	0.10
	Axial forces	Lognormal	1.00	0.05
Shell models	Shear forces	Lognormal	1.00	0.10
	Moments	Lognormal	1.00	0.20
	Forces	Lognormal	1.00	0.10

4.2.2 Examples

Table 6 exemplifies values of γ_{Sd} obtained using Eq. (25) for different pairs ($\mu_{\theta E}$, $V_{\theta E}$), considering the case $\alpha = -0.28$ (i.e. assuming that the variable θ_E is non dominant, as indicated in Table 2). For models with $\mu_{\theta E} < 1.00$ and small $V_{\theta E}$, the γ_{Sd} -factor is less than 1.00, which means that those models are globally conservative.

Table 6 Partial factors for model uncertainties of load effects, γ_{Sd} ($\alpha = -0.28$)

$\mu_{\theta E}$	$V_{\theta E}$	Consequences		
		Low (β = 3.30)	Medium (β = 3.80)	High (β = 4.30)
0.90	0.05	0.94	0.95	0.95
	0.10	0.98	1.00	1.01
	0.15	1.02	1.04	1.07
	0.20	1.06	1.09	1.12
1.00	0.05	1.05	1.05	1.06
	0.10	1.09	1.11	1.12
	0.15	1.14	1.16	1.18
	0.20	1.18	1.21	1.25
1.10	0.05	1.15	1.16	1.17
	0.10	1.20	1.22	1.23
	0.15	1.25	1.28	1.30
	0.20	1.30	1.33	1.37

In most cases, γ_{Sd}-factors will be between 1.05 and 1.15, as recommended in EN 1990 [2], Table A1.2 (B).

4.3 γ_F - factors

Once defined the factors γ_f and γ_{Sd}, the design value E_d for a given limit state can be evaluated according to Eurocode 0 [2], as follows (see Eq. (11)):

$$E_d = \gamma_{Sd} \cdot E(\gamma_{f1} F_{1k}, \gamma_{f2} F_{2k}, \ldots) \qquad (27)$$

where F_1, F_2,... represent actions and other basic variables relevant for the load effect E. As an alternative, E_d can be computed by:

$$E_d = E(\gamma_{F1} F_{1k}, \gamma_{F2} F_{2k}, \ldots) \qquad (28)$$

where the factors γ_{Fi} are given by:

$$\gamma_{Fi} = \gamma_{Sd}\, \gamma_{fi} \qquad (29)$$

Clearly, Equations (27) and (28) give the same result only if the load effect E is a linear function of the basic variables F_i. However, according to [2], both equations are acceptable.

In short, the factors $\gamma_{Fi} = \gamma_{Sd}\, \gamma_{fi}$ intends to take into account all uncertainties in the actions side and are comparable to the factors for actions specified in Eurocodes.

5 Partial factors for resistances

5.1 γ_m - factors

5.1.1 Resistances with normal distribution

Using Eq. (6) and the inverse of the Normal distribution, the partial factor for resistances with Normal distribution and coefficient of variation V is given by:

$$\gamma_m = \frac{1+\Phi^{-1}(p)V}{1-\alpha\beta V} \qquad (30)$$

where p is the fractile used to quantify the characteristic value of the resistance under consideration. For resistances, the 0.05 fractile is frequently used. Thus, considering that $\Phi^{-1}(0.05) = -1.645$, for this particular fractile the partial factor is:

$$\gamma_m = \frac{1-1.645\,V}{1-\alpha\beta V} \qquad (31)$$

5.1.2 Resistances with lognormal distribution

Adopting a procedure similar to that used for actions with Lognormal distribution, the following expression is derived:

$$\gamma_m = \exp\left(\sqrt{\ln(1+V^2)}\left(\alpha\beta+\Phi^{-1}(p)\right)\right) \qquad (32)$$

5.1.3 Resistances with Weibull distribution

Consider now a resistance variable with distribution $Wb^{(\in,u,k)}$, whose domain is $x \geq \in$. The frequent case $\in = 0$ will be considered in the following. Using the inverse of the Weibull distribution (see Annex A), the Eq. (6) leads to the following partial factor:

$$\gamma_m = \left(\frac{\ln(1-p)}{\ln(1-\Phi(-\alpha\beta))}\right)^{\frac{1}{k}} \qquad (33)$$

The parameter k is a shape parameter which is directly related to the coefficient of variation. For the case $\in = 0$, k depends solely on the coefficient of variation V. Table 7 shows values of k corresponding to some values of V.

Table 7 Values of the parameter k corresponding to different coefficients of variation — Weibull distribution

V	k
0.05	24.95
0.10	12.15
0.15	7.91
0.20	5.80
0.25	4.54
0.30	3.71

5.1.4 Examples

Table 8 shows γ_m-factors obtained using the equations (31), (32) and (33), considering in all cases $\alpha = 0.80$ and $p = 0.05$. For the Normal distribution, the partial factors were computed considering a maximum coefficient of variation of 0.25. Above this value, modelling resistances with Normal distribution is questionable, since the probability of obtaining negative resistances becomes non negligible. In effect, for a Normal variable X with coefficient of variation V, $P(X < 0) = \Phi(-1/V)$ and for $V = 0.25$, $P(X < 0) = 3.2 \times 10^{-5}$, which is of the order of magnitude of the usual failure probabilities and consequently is not negligible.

Table 8 Partial factors for resistances, γ_m ($\alpha = 0.80$, $p = 0.05$)

Model	V	Consequences		
		Low ($\beta = 3.3$)	Medium ($\beta = 3.8$)	High ($\beta = 4.3$)
Normal	0.05	1.06	1.08	1.11
	0.10	1.14	1.20	1.27
	0.15	1.25	1.38	1.56
	0.20	1.42	1.71	2.15
	0.25	1.73	2.45	4.21
Lognormal	0.05	1.05	1.07	1.09
	0.10	1.10	1.15	1.20
	0.15	1.16	1.23	1.31
	0.20	1.22	1.32	1.43
	0.25	1.28	1.41	1.56
	0.30	1.34	1.51	1.69
Weibull	0.05	1.11	1.16	1.23
	0.10	1.23	1.36	1.53
	0.15	1.37	1.61	1.92
	0.20	1.54	1.92	2.44
	0.25	1.74	2.29	3.12
	0.30	1.97	2.76	4.03

Observing Table 8, it is seen that the Weibull model is the most conservative (in the sense that it leads to higher partial factors), and the Lognormal is the least conservative. This result is a direct consequence of the weight of the left tails, being heavier in the Weibull model and lighter in the Lognormal model.

Moreover, the difference between models is more significant for high coefficients of variation, similarly to what was observed regarding partial factors for actions. As a result, when choosing a probabilistic model for a resistance variable, more attention must be paid if a variable has high coefficient of variation.

Figure 3 plots the results shown in Table 8 for the Reliability Class RC2 ($\beta = 3.8$). As it is seen, for coefficients of variation greater than 0.23, the Normal model is more conservative than the Weibull model, which shows that above that coefficient of variation the left tail of the Normal model becomes heavier comparatively with the Weibull model. Nevertheless, as mentioned above, the Normal model should not be used in modelling resistances with high coefficients of variation.

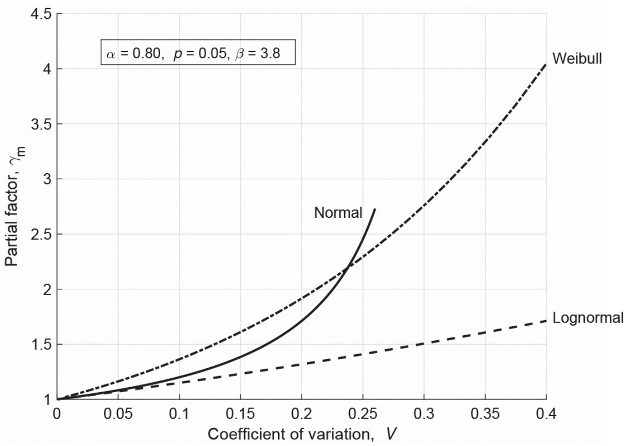

Figure 3 Partial factors for resistances

5.2 γ_{Rd}-factors

5.2.1 General expression

The variable θ_R is in general modelled by a Lognormal distribution [13, 14]. Thus, using the expression for the inverse of the Lognormal model, Eq. (16) yields:

$$\gamma_{Rd} = \frac{1}{\dfrac{\mu_{\theta R}}{\sqrt{1+V_{\theta R}^2}} \cdot \exp\left(-\alpha_R \beta \sqrt{\ln\left(1+V_{\theta R}^2\right)}\right)} \tag{34}$$

where $\mu_{\theta R}$ and $V_{\theta R}$ are the mean and coefficient of variation of θ_R. If $V_{\theta R} \leq 0.20$, Eq. (34) can be approximated by:

$$\gamma_{Rd} \approx \frac{1}{\mu_{\theta R} \cdot \exp\left(-\alpha_R \beta V_{\theta R}\right)} \tag{35}$$

The mean $\mu_{\theta R}$ and the coefficient of variation $V_{\theta R}$ should be chosen carefully case-by-case, since they depend on the accuracy and precision of the resistance model in use.

Table 9 shows the recommendations of JCSS [13] concerning the variable θ_R. As shown, the recommended mean for θ_R is higher than unity in all cases, which reflects the perception that resistance models are generally conservative. The values in Table 9 should be interpreted as mere indication and should be adjusted on a case-by-case basis, depending on the confidence in the resistance model being used.

In a recent study [19], based on a survey involving several hundreds of laboratory tests, the authors proposed for concrete structures the models shown in Table 10. The proposed statistical parameters applies to concrete structures not affected by corrosion, designed according to Eurocode 2 [20]. As observed, for members requiring design shear reinforcement, the resistance model proposed in [20], based on Mörsch truss without any correcting factor, is rather conservative.

Table 9 Probabilistic models for the variable θ_R, as recommended by JCSS [13]

Structural material	Resistance type	Distribution	$\mu_{\theta R}$	$V_{\theta R}$
Structural steel	Bending capacity	Lognormal	1.00	0.05
	Shear capacity	Lognormal	1.00	0.05
	Welded connection capacity	Lognormal	1.15	0.15
	Bolted connection capacity	Lognormal	1.25	0.15
Reinforced concrete	Bending capacity	Lognormal	1.20	0.15
	Shear capacity	Lognormal	1.40	0.25

Table 10 Probabilistic models for θ_R for sound concrete structures [19]

Resistance type	Distribution	$\mu_{\theta R}$	$V_{\theta R}$
Axial compression without effects of buckling	Lognormal	1.00	0.05
Bending	Lognormal	1.075	0.075
Shear in members without special shear reinforcement	Lognormal	1.00	0.20
Shear in members with lightly shear reinforcement[a]	Lognormal	1.825	0.25
Shear in members with moderately shear reinforcement[b]	Lognormal	1.275	0.20

[a] $\rho_w f_{yw} < 1$ MPa [b] $1 < \rho_w f_{yw} \leq 2$ MPa

5.2.2 Examples

Table 11 exemplifies values of γ_{Rd} obtained using Eq. (34) for different pairs ($\mu_{\theta R}$, $V_{\theta R}$). As observed, in many cases γ_{Rd} is less than 1.00. In those cases the random errors in the resistance model (as reflected in $V_{\theta R}$) were counterbalanced by the favourable systematic errors (as reflected in $\mu_{\theta R}$ greater than 1.00).

5.3 γ_M-factors

Once γ_m and γ_{Rd}-factors are defined, the design value R_d for a given limit state can be evaluated according to Eurocode 0 [2], as follows (see Eq. (15)):

$$R_d = \frac{1}{\gamma_{Rd}} \cdot R\left(\frac{f_{1k}}{\gamma_{m1}}, \frac{f_{2k}}{\gamma_{m2}}, \ldots\right) \tag{36}$$

where $f_1, f_2,...,$ represent resistances and other basic variables relevant for the resistance R. As an alternative, R_d can be computed by:

$$R_d = R\left(\frac{f_{1k}}{\gamma_{M1}}, \frac{f_{2k}}{\gamma_{M2}}, \ldots\right) \tag{37}$$

where the factors γ_{Mi} are given by:

$$\gamma_{Mi} = \gamma_{Rd}\,\gamma_{mi} \tag{38}$$

Clearly, Equations (36) and (37) give the same result only if the resistance R is a linear function in the basic variables f_i. However, according to [2], both alternatives are acceptable. In short, the factors $\gamma_{Mi} = \gamma_{Rd}\,\gamma_{mi}$ intends to take into account all uncertainties in the resistance side and are comparable to the factors for resistances specified in Eurocodes.

Table11 Partial factors for uncertainties in resistance models, γ_{Rd} ($\alpha = 0.32$)

$\mu_{\theta R}$	$V_{\theta R}$	Consequences		
		Low ($\beta = 3.30$)	Medium ($\beta = 3.80$)	High ($\beta = 4.3$)
1.00	0.05	1.06	1.06	1.07
	0.10	1.12	1.13	1.15
	0.15	1.18	1.21	1.24
	0.20	1.26	1.30	1.34
1.10	0.05	0.96	0.97	0.98
	0.10	1.02	1.03	1.05
	0.15	1.08	1.10	1.13
	0.20	1.15	1.18	1.22
1.20	0.05	0.88	0.89	0.89
	0.10	0.93	0.95	0.96
	0.15	0.99	1.01	1.04
	0.20	1.05	1.08	1.12
1.30	0.05	0.81	0.82	0.83
	0.10	0.86	0.87	0.89
	0.15	0.91	0.93	0.96
	0.20	0.97	1.00	1.03
1.40	0.05	0.75	0.76	0.77
	0.10	0.80	0.81	0.82
	0.15	0.85	0.87	0.89
	0.20	0.90	0.93	0.96

6 Conclusions

Expressions for the determination of partial factors based on the concept of FORM design value were presented. Two groups of partial factors were distinguished, namely, partial factors for basic variables representing actions and material resistances (γ_f and γ_m), and partial factors for variables representing model uncertainties (γ_{Sd} and γ_{Rd}).

Regarding the first ones, the expressions presented show that those factors depend on:

- probabilistic model used to describe uncertainty in the basic variable;
- coefficient of variation of the variable;
- fractile implicit on the characteristic value used to compute the design value;
- importance of the variable in the limit state under consideration (measured by the respective sensitivity factor);
- target reliability index.

The choice of the type of probabilistic model influences significantly the partial factors and this influence rises as the coefficient of variation increases. This means that more attention must be paid when choosing a probabilistic model for a variable with high coefficient of variation.

The expressions presented were exemplified using the coefficients of sensitivity recommended in [2] for dominant actions and dominant resistances, respectively $\alpha = -0.70$ and $\alpha = 0.80$. The cases for non-dominant variables ($\alpha = -0.28$ and $\alpha = 0.32$) can lead to partial factors γ_f or γ_m less than 1.0. In fact, the characteristic value of a basic variable incorporates already some safety margin, because it is a relatively small fractile (0.05 in the case of materials) or a relatively large one (0.95 in the case of actions). Obtaining safety factors less than 1.0 means that the safety incorporated in characteristic values is far sufficient for the intended reliability. Note that this is in agreement with the use of the ψ_0 factor connected to the partial factors method, in which ψ_0 multiplied by the safety factor gives a value frequently less than 1.0.

Regarding partial safety factors for model uncertainties (γ_{Sd} and γ_{Rd}), they have a slightly different nature when compared to γ_f and γ_m-factors, because they coincide with the design values of the variables accounting for model uncertainties, θ_E and θ_R, that is, they incorporate both characteristic values and safety factors.

Expressions presented for γ_{Sd} and γ_{Rd} show that these factors depend essencialy on:

- mean and coefficient of the corresponding variation of variable θ;
- importance of the variable θ in the limit state under consideration (measured by the respective sensitivity factor);
- target reliability index.

The mean and coefficient of variation of the variables accounting for model uncertainties have a well defined meaning: the first one constitutes a measure of the model accuracy, that is, its ability to predict values with small systematic errors; the second one constitutes a measure of the model precision, that is, its ability to predict values with small random errors (low variability).

Recent studies have dealt with the accuracy and precision of the resistance models specified in Eurocode 2 [20] for reinforced concrete members. Similar studies are desirable regarding action models and structural models as well.

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Annex

Probabilistic models used in the article

Normal	$X \sim N(\mu_x, \sigma_x)$
PDF	$f_X(x \mid \mu_X, \sigma_X) = \frac{1}{\sqrt{2\pi}\sigma_X}\exp\left[-\frac{1}{2}\left(\frac{x-\mu_X}{\sigma_X}\right)^2\right] \qquad (-\infty < x < +\infty)$
Moments	$\mu_X = \mu_X \qquad \sigma_X^2 = (\sigma_X)^2$
CDF	$F_X(x) = \Phi\left(\frac{x-\mu_X}{\sigma_X}\right)$
Inverse	$F_X^{-1}(p) = \mu_X + \sigma_X \Phi^{-1}(p)$

Lognormal	$X \sim \mathrm{LN}(a,b)$
PDF	$f_x(x \mid a,b) = \frac{1}{\sqrt{2\pi}bx}\exp\left[-\frac{1}{2}\left(\frac{\ln x - a}{b}\right)^2\right] \qquad (0 < x < +\infty)$
Moments	$\mu_X = e^{a+\frac{1}{2}b^2} \qquad \sigma_X^2 = \left(e^{b^2}-1\right)e^{2a+b^2}$
CDF	$F_X(x) = \Phi\left(\frac{\ln x - a}{b}\right)$
Inverse	$F_X^{-1}(p) = \frac{\mu_X}{\sqrt{1+V_X^2}} \cdot \exp\left(\Phi^{-1}(p) \cdot \sqrt{\ln\left(1+V_X^2\right)}\right) \qquad V_X = \frac{\sigma_X}{\mu_X}$ For $V_x \leq 0.20$, the following approximation is admissible: $F_X^{-1}(p) = \mu_X \cdot \exp\left(\Phi^{-1}(p) \cdot V_X\right)$

Gumbel	$X \sim Gb(u,\alpha)$
PDF	$f_X(x \mid u,\alpha) = \alpha \exp\left\{-\alpha(x-u) - \exp\left[-\alpha(x-u)\right]\right\} \qquad (-\infty < x < +\infty)$
Moments	$\mu_X = u + \frac{\gamma}{\alpha} \qquad \sigma_X^2 = \left(\frac{\pi}{\sqrt{6}\alpha}\right)^2 \qquad (\gamma \approx 0.57722)$
CDF	$F_X(x) = \exp\left\{-\exp\left[-\alpha(x-u)\right]\right\}$
Inverse	$F_X^{-1}(p) = \mu_X - 0.78\left[0.58 + \ln(-\ln p)\right]\sigma_X$

Fréchet	$X \sim Fr(u,k)$
PDF	$f_X(x \mid u,k) = \frac{k}{u}\left(\frac{u}{x}\right)^{k+1} \exp\left[-\left(\frac{u}{x}\right)^{k}\right] \qquad (0 < x < +\infty)$
Moments	$u_X = u\Gamma\left(1-\frac{1}{k}\right) \qquad \sigma_x^2 = u^2\left[\Gamma\left(1-\frac{2}{k}\right)-\Gamma^2\left(1-\frac{1}{k}\right)\right] \qquad (k > 2)$
CDF	$F_X(x \mid u,k) = \exp\left[-\left(\frac{u}{x}\right)^{k}\right]$
Inverse	$F_X^{-1}(p) = \frac{u}{(-\ln p)^{\frac{1}{k}}}$

Weibull	$X \sim Wb(\in,u,k)$
PDF	$f_X(x \mid \in,u,k) = \frac{k}{u-\in}\left(\frac{x-\in}{u-\in}\right)^{k-1} \exp\left[-\left(\frac{x-\in}{u-\in}\right)^{k}\right] \qquad (\in < x < +\infty)$
Moments	$u_X = \in + (u-\in)\cdot\Gamma(1+1/k) \qquad \sigma_x^2 = (u-\in)^2\left[\Gamma\left(1+\frac{2}{k}\right)-\Gamma^2\left(1+\frac{1}{k}\right)\right]$
CDF	$F_X(x \mid \in,u,k) = 1-\exp\left[-\left(\frac{x-\in}{u-\in}\right)^{k}\right]$
Inverse	$F_X^{-1}(p) = \in + (u-\in)\left[-\ln(1-p)\right]^{\frac{1}{k}}$